I $f(x) = \begin{cases} x+1 & \text{if } x<1 \\ 3-(x-2)^2 & \text{if } x \geq 1 \end{cases}$

a) $f(1) = 3-(-1)^2$

$3-1=2$ ← ∴ f is cont at $x=2$

$\lim_{x\to 1^-} x+1 = 2$

$\lim_{x\to 1^+} 3-(x-2)^2 = 2$

$\lim_{x\to 1} f(x) = 2$

b) $\lim_{x\to 1} \frac{f(x)-2}{x-1}$

$\lim_{x\to 1^-} \frac{x+1-2}{x-1}$

$\lim_{x\to 1^-} \frac{x-1}{x-1} = 1$

$\lim_{x\to 1^+} \frac{3-(x-2)^2-2}{x-1}$

$\lim_{x\to 1^+} \frac{1-(x^2-4x+4)}{x-1}$

$\lim_{x\to 1^+} \frac{-(x^2-4x+3)}{x-1}$

$\lim_{x\to 1^+} \frac{-(x-1)(x-3)}{x-1}$

$\lim_{x\to 1^+} -(-2) = 2$

∴ limit DNE

(D) ~~(E)~~

Since limit is left of Der, Der DNE at $x=1$

II $f(x) = x + \frac{1}{x}$

$\lim_{h\to 0} \frac{x+h+\frac{1}{x+h} - x - \frac{1}{x}}{h}$

$\lim_{h\to 0} \frac{1}{h}\left[\frac{xh(x+h) + x - (x+h)}{x(x+h)}\right]$

$\lim_{h\to 0} \frac{1}{h}\left[\frac{x^2h + xh^2 + x - x - h}{x(x+h)}\right]$

$\lim_{h\to 0} \frac{x^2 + xh - 1}{x(x+h)} = \frac{x^2-1}{x^2}$

III $f(x) = x^4 - 4x^3 + 10$

(A) $f'(x) = 4x^3 - 12x^2$

$4x^2(x-3)$

$f''(x) = 12x^2 - 24x$

$12x(x-2)$

(B)

f': − 0 − 0 +, critical numbers 0, 3

f'': + 0 − 0 +, inflection points 0, 2

(C) $f(0) = 10$ (0, 10) Lady PI

$f(3) = 81 - 108 + 10 = -17$

(3, −17) min

(D) Yes, one at $x=2$

$16 - 32 + 10$

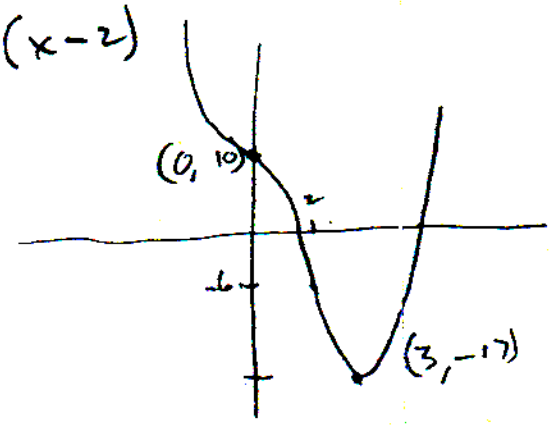
(2, −6) PI

which is not critical value

(E)

f increases for $x > 3$

f concave up for $x < 0$ or $x > 2$



IV

a) $\lim_{x\to 0} \frac{\sqrt{x^2+9}-3}{x^2} \cdot \frac{\sqrt{}+3}{\sqrt{}+3}$

$\lim_{x\to 0} \frac{x^2+9-9}{x^2(\sqrt{x^2+9}+3)} = \frac{1}{3+3} = \frac{1}{6}$

b) $\lim_{x\to 0} \left[\frac{3+2x}{x+5x^2} - \frac{3}{x}\right]$ (common denominator $x(1+5x)$)

$\lim_{x\to 0} \left[\frac{3+2x-3(1+5x)}{x(1+5x)}\right]$

$\lim_{x\to 0} \frac{3+2x-3-15x}{x(1+5x)}$

$\lim_{x\to 0} \frac{-13}{1+5x} = -13$

c) $\lim_{x\to 1} \frac{x-1}{\sqrt[3]{x}-1}$ $\quad (x+1)-2$

$\lim_{x\to 1} \frac{(x^{\frac{1}{3}}-1)(x^{\frac{2}{3}}+x^{\frac{1}{3}}+1)}{x^{\frac{1}{3}}-1} = 3$

d) $\lim_{x\to -1} \frac{x+1}{x^2+1} = \frac{0}{2} = 0$

Extra:

This is a Proof

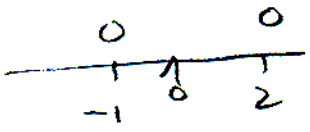
$f(2) = 0$

$\lim_{x\to 2^-} |x+1|\operatorname{sgn}(x-2)$

$\lim_{x\to 2^-} (x+1)(-1) = -3$

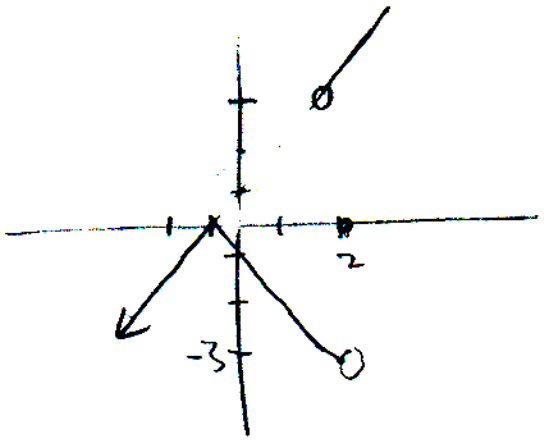
$\lim_{x\to 2^+} (x+1)(1) = 3$

$\therefore \lim_{x\to 2} f(x)$ DNE $\therefore f$ not cont at $x=2$



This is not a Proof

$$f(x) = \begin{cases} x+1 & \text{if } x<-1 \\ 0 & \text{if } x=-1 \\ -x-1 & \text{if } -1<x<2 \\ 0 & \text{if } x=2 \\ x+1 & \text{if } x>2 \end{cases}$$



I $-(x+1)(-1)$

III $(x+1)(-1)$

91

	2	5	7
100	1	0	1
90	7	5	12
80	8	9	17
70	3	4	7
60	2	2	4
59	0	2	2

100 → 58

1785 / 24 = 85.0

1762 / 22 = 80.1

82.5

3/24/2000

	4	5	7
100	1	2	3
90	3	8	11
80	4	3	7
70	8	4	12
60	2	3	5
59	2	1	3

103 → 36

1612 / 21 = 76.8

1752 / 21 = 83.4

42

80.1

Excellent exam.

This shortened #-of days didn't seem to hurt any except those absent a day or two.

Honors Analysis

Name _____

Copy the original problem.
Convince *me* that **you** understand the concept.

Per _____ Date 1991 _____

CHAPTER #3

2000

I Given: $f(x)=\begin{cases} x+1 & \text{if } x<1 \\ 3-(x-2)^2 & \text{if } x\geq 1 \end{cases}$ (tot 20 pts)

a) Prove $f(x)$ is continuous at $x=1$.

b) Prove $f'(1)$ does not exist.

II Using the second definition of the derivative ($\lim_{h\to 0}$ method), find $F'(x)$ when $F(x)=x+\frac{1}{x}$ (tot 20 pts)

III Given: $F(x)=x^4-4x^3+10$ (tot 40 pts)

a) Find $F'(x)$ and $F''(x)$.

b) Draw and properly label areas of interest on number lines for each $F'(x)$ and $F''(x)$.

c) Find the coordinates of the critical points of F. Identify them as max, min or PI. Justify your responses.

d) Are there any PI which are not at critical values of x? Where?

e) Determine and state the interval(s) where:

1) F is increasing. 2) F is concave up.

f) Sketch $F(x)$.

IV Determine the following limits: (5 pts ea)

a) $\lim_{x\to 0}\frac{\sqrt{x^2+9}-3}{x^2}$

b) $\lim_{x\to 0}\left(\frac{3+2x}{x+5x^2}-\frac{3}{x}\right)$

c) $\lim_{x\to 1}\frac{x-1}{\sqrt[3]{x}-1}$

d) $\lim_{x\to -1}\frac{x+1}{x^2+1}$

EXTRA CREDIT 5 pts

Sketch F. Prove F is or is not continuous at $x=2$ when $F(x)=|x+1|\operatorname{sgn}(x-2)$

I

a) $f(x) = (4x^3 - 5x)^2$

$16x^6 - 40x^4 + 25x^2$

$f' = 96x^5 - 160x^3 + 50x$

b) $f(x) = x^\pi + 3$

$f'(x) = \pi x^{\pi-1}$

c) $f(x) = 4^2 - 1$

$f'(x) = 0$

error w/ reason -2

else -3

II $f(x) = (2x-3)^2 = 4x^2 - 12x + 9$

$f(x+h) = 4(x+h)^2 - 12(x+h) + 9$

$\lim_{h\to 0} \frac{1}{h}\left[4x^2 + 8hx + 4h^2 - 12x - 12h + 9 - 4x^2 + 12x - 9\right]$

$\lim_{h\to 0} \frac{1}{h}\left[h(8x + 4h - 12)\right]$

$\lim_{h\to 0} 8x + 4h - 12 = f'(x)$

III $f(x) = \begin{cases} x^2 & x \le 1 \\ 2x - 1 & x > 1 \end{cases}$

a) $\lim_{x\to a} f(x) = f(a)$

$f(1) = 1$ (I)

$\lim_{x\to 1^-} x^2 = 1$ (II) $= 1 =$ (III)

$\lim_{x\to 1^+} 2x - 1 = 1$

b) $\lim_{x\to 1} \frac{f(x) - 1}{x - 1}$

$\lim_{x\to 1^-} \frac{x^2 - 1}{x - 1}$ $\quad \lim_{x\to 1^+} \frac{2x - 2}{x - 1}$

$\lim_{x\to 1} x + 1 = 2$ $\quad \lim_{x\to 1^+} 2$

$L = 2$

$\lim_{x\to 1} \frac{f(x) - 1}{x - 1} = 2$

The limit Does exist at $x = 1$

IV $f(x) = x^4 - 4x^3 = x^3(x - 4)$

a) $f'(x) = 4x^3 - 12x^2$

$4x^2(x - 3)$

f': $-$ (0) $-$ (3) $+$

$f''(x) = 12x^2 - 24x$

$12x(x - 2)$

f'': $+$ (0) $-$ (2) $+$

b) $f(0) = 0$ $\quad 0, 0$ LAZY PI

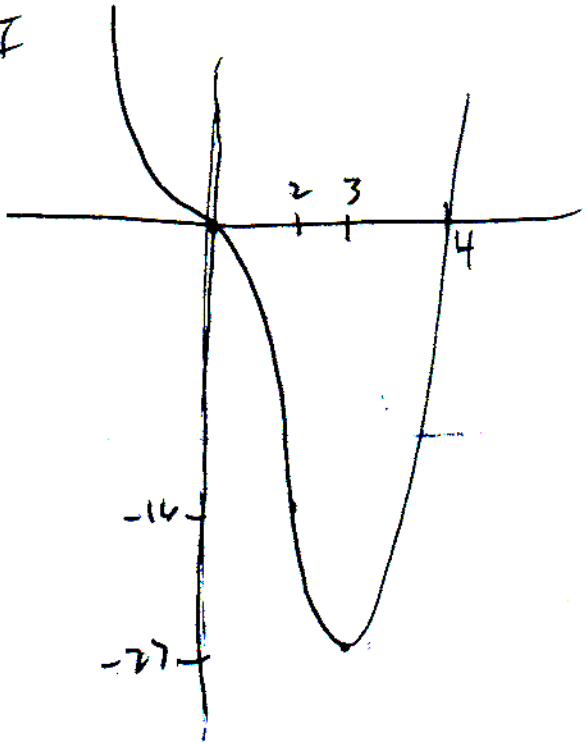
$f(3) = -27$ $\quad (3, -27)$ min

c) $f(2) = 8(-2) = -16$

$(2, -16)$ also PI

d) f concave up for $x < 0$ or $x > 2$

e) f decreases for $x < 3$; $x \neq 0$



V

a) $\lim_{x\to 2} \frac{\sqrt{x^2+5}-3}{x-2}\cdot\frac{\sqrt{}+3}{\sqrt{}+3}$

$\lim_{x\to 2} \frac{x^2+5-9}{(x-2)(\sqrt{x^2+5}+3)}$

$\lim_{x\to 2} \frac{x^2-4}{(x-2)(\sqrt{\quad}+3)}$

$\lim_{x\to 2} \frac{x+2}{\sqrt{x^2+5}+3} = \frac{4}{3+3} = \frac{4}{6} = \frac{2}{3}.$

b) $\lim_{x\to 0} \sqrt{1+\frac{1}{x}} - \sqrt{\frac{1}{x}}$

$\lim_{x\to 0} \frac{\sqrt{x+1}-1}{\sqrt{x}}\cdot\frac{\sqrt{}+1}{\sqrt{}+1}$

$\lim_{x\to 0} \frac{x+1-1}{\sqrt{x}(\sqrt{x+1}+1)}$

$\lim_{x\to 0} \frac{\sqrt{x}}{\sqrt{x+1}+1} = \frac{0}{1+1} = 0$

c) $\lim_{x\to 1} \left[\frac{1}{1-x^3} - \frac{1}{3(1-x)}\right]$

$\lim_{x\to 1} \left[\frac{3}{3(1-x)(1+x+x^2)} - \frac{1+x+x^2}{3(1-x)(1+x+x^2)}\right]$

$\lim_{x\to 1} \frac{-(x^2+x-2)}{3(1-x)(1+x+x^2)}$

$\lim_{x\to 1} \frac{\cancel{-}(x+2)\cancel{(x-1)}}{3\cancel{(1-x)}(1+x+x^2)}$ $\quad \frac{\cancel{3}}{\cancel{3}(3)} = \frac{1}{3}$

Extra

$f(x) = ax^3+bx^2$ (1, 2)

$f(1) = a+b \overset{\text{Set}}{=} 2$

Also ....

$f'(x) = 3ax^2+2bx$

$f''(x) = 6ax+2b$

$f''(1) = 6a+2b \overset{\text{Set}}{=} 0$ because (1, 2) is P.I.'. $x=1$ but $f''=$ zero

So ....

$3a+b=0$

$a+b=2$

$2a = -2$

$a=-1 \quad \therefore b=3$

	4	5	T
100	3	3	
90	7	11	
80	2	4	
70	4	5	
60	0	1	
54	2	0	
	1522 / 18	2036 / 24	50

103

84.6 84.8 84.7

Great Question and Great Results!!
They might actually understand this!!
Imagine That!

Honors Analysis

Name _____

Per _____ Date _____

Copy original problem.

Convince *me* that **you** understand the concept!

2001

Chapter 3 Exam

I Given $f(x)$, find $f'(x)$: (5 pts ea)

a) $(4x^3 - 5x)^2$ b) $x^{\pi} + 3$ c) $4^2 - 1$

II Using the "definition of the derivative" which yields a function, find $f'(x)$: $f(x) = (2x-3)^2$ (15 pts)

III Given $f(x) = \begin{cases} x^2 & ; \ x \le 1 \\ 2x-1 & ; \ x > 1 \end{cases}$ (15 pts ea)

a) ***Prove*** $f(x)$ is or is not continuous at $x = 1$. b) ***Prove*** $f'(1)$ does or does not exist.

IV Given: $f(x) = x^4 - 4x^3$ 25 pts tot)

a) Find $f'(x)$ and $f''(x)$. Draw appropriate "number line" graphs.

b) Find the *coordinates* of the critical points. Identify them as max, min or P.I.

c) Determine the *coordinates* of any PI which are NOT critical values.

d) Specifically *state the interval*(s) where $f(x)$ is concave up.

e) Specifically *state the interval*(s) where $f(x)$ is decreasing.

f) Sketch the graph of $f(x)$.

V Determine the following limits: (5 pts ea)

a) $\lim_{x \to 2} \dfrac{\sqrt{x^2+5} - 3}{x-2}$ b) $\lim_{x \to 0} \sqrt{1 + \dfrac{1}{x}} - \sqrt{\dfrac{1}{x}}$ c) $\lim_{x \to 1} \left[\dfrac{1}{1-x^3} - \dfrac{1}{3(1-x)} \right]$

Extra Credit ---------- 5 pts ----------

Given $f(x) = ax^3 + bx^2$.

Determine a and b so the graph of $f(x)$ will have a point of inflection at (1, 2).

I $f(x) =$ $\begin{cases} \frac{2}{3}x+1 & -3 < x < 1 \\ 3 & x = -1 \\ -x-2 & -1 < x < 1 \\ x-4 & 1 < x < 3 \\ -\frac{4}{3}x+3 & 3 \le x < 5 \end{cases}$

use def of der INSTEAD of def of continuity -5 plus any other errors.

a) $x = -1$

$f(-1) = 3$

$\lim_{x \to -1^-} \frac{2}{3}x+1 = \frac{1}{3}$ ← (L) ≠ (R) ← Limit DNE

$\lim_{x \to -1^+} -x-2 = -1$

f not cont at $x = -1$

b) 4pts $f(1)$ DNE ∴ f not cont $x = 1$

c) $f(3) = -\frac{4}{3}(3)+3 = -1$ ← =

$\lim_{x \to 3^-} x-4 = -1$] $\lim_{x \to 3} f(x) = -1$

$\lim_{x \to 3^+} -\frac{4}{3}(3)+3 = -1$

∴ f is cont at $x = 3$

II $f(x) = x^2+1$

$$\lim_{h \to 0} \frac{(x+h)^2+1-(x^2+1)}{h}$$

$$\lim_{h \to 0} \frac{x^2+2xh+h^2+1-x^2-1}{h}$$

$$\lim_{h \to 0} \frac{h(2x+h)}{h} = 2x$$

III $f(x) = |x^2-4|$ no def for $x = -2$

$$\lim_{x \to 2} \frac{|x^2-4|-0}{x-2}$$

This limit DNE ∴ (L) ≠ (R)

$$\lim_{x \to 2^-} \frac{-(x^2-4)}{x-2}$$

$$\lim_{x \to 2^-} \frac{-(x-2)(x+2)}{x-2} = -4$$

$$\lim_{x \to 2^+} \frac{x^2-4}{x-2}$$

$$\lim_{x \to 2^+} \frac{(x+2)(x-2)}{(x-2)}$$

4

(L) ≠ R

Since the limit is the def of the der, the derivative doesn't exist.

V A) $\lim_{x \to 1} \frac{x+1}{x^2-1}$

$\lim_{x \to 1} \frac{x+1}{(x+1)(x-1)}$

$\lim_{x \to 1} \frac{1}{x-1}$

No limit

$\lim_{x \to 1^+} > 0$

$\lim_{x \to 1^-} < 0$

B) $\lim_{x \to 2} \frac{x^3-8}{x-2}$

$\lim_{x \to 2} \frac{(x-2)(x^2+2x+4)}{x-2}$

$4+4+4$

12

C) $\lim_{x \to 3} \frac{\sqrt{x+1}}{\sqrt{x-5}}$

No Limit, CAN Not Approach

IV $f(x) = 4x^3 - 12x^2 + 8$

a) $4 \quad -12 \quad 0 \quad 8$

1 | $4 \quad -8 \quad -8 \quad 0$

$1 \quad -2 \quad -2$

$x^2 - 2x - 2$

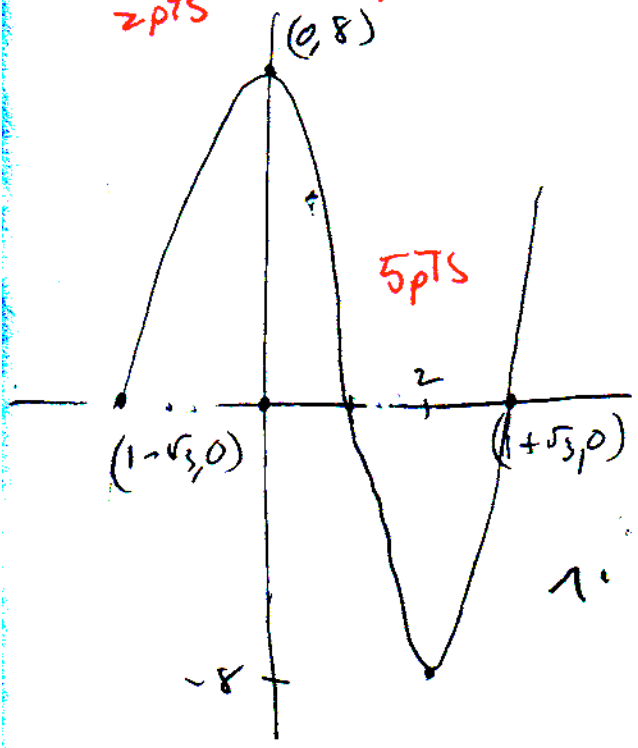
$x = \frac{2 \pm \sqrt{4+8}}{2}$

$\frac{2 \pm 2\sqrt{3}}{2}$ 3pts

$1 \pm \sqrt{3}$

$(1, 0) \quad (1-\sqrt{3}, 0) \quad (1+\sqrt{3}, 0)$

2pts 2pts 2pts



b) $f' = 12x^2 - 24x$

$12x(x-2)$

5pts

f'	+	0	−	0	+
	↑		↓		↑
		0		2	

2pts f is increasing for $x < 0$ or $x > 2$

2pts f is decreasing for $0 < x < 2$

c) $f(0) = 8$ $(0, 8)$ max 3pts

$f(x) = 32 - 48 + 8$ $(2, -8)$ min 3pts

d) $f'' = 24x - 24$

$24(x-1)$

5pts

f''	−	0	+
	↓		↑
		1	

2pts PI at $(1, 0)$ changes concavity.

V a) $\lim_{x \to 1} \frac{x+1}{x^2-1}$

$\lim_{x \to 1} \frac{x+1}{(x-1)(x+1)}$

$\lim_{x \to 1^-} \frac{1}{x-1} \Rightarrow -\infty$ DNE

$\lim_{x \to 1^+} \frac{1}{x+1} \Rightarrow +\infty$

b) $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$

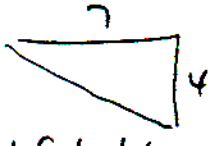
$\lim_{x \to 2} \frac{(x-2)(x^2+2x+4)}{x-2}$

$4 + 4 + 4$

12

c) $\lim_{x \to 3} \frac{\sqrt{x-1}}{\sqrt{x-5}}$ DNE CANNOT Approach.

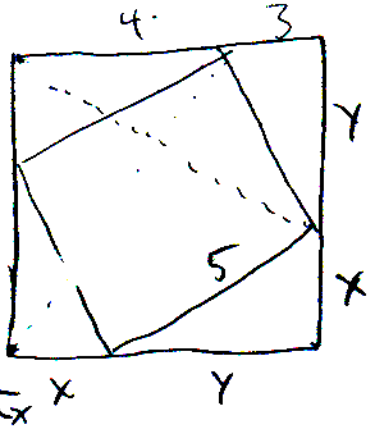
EXTRA.



$49 + 16$

$\sqrt{65}$

Longest from one vertex to the other.



$x + y = 7 \quad y = 7 - x$

$x^2 + y^2 = 25$

$x^2 + (7-x)^2 = 25$

$x^2 + x^2 - 14x + 49 = 25$

$2x^2 - 14x + 24$

$x^2 - 7x + 12$

$(x-3)(x-4) = 0$

3rd

100	5	0	5
90	2	3	5
80	3	10	13
70	6	4	9
60	2	5	7
50	1	6	7

104

39

02

1519 / 18 = 84.4

2012 / 28 = 71.9

3531 / 46 = 76.8

a very easy exam.

The limits should be changed so all 3 ~~exist~~ do.

That forces them to go figure out how to do it.

I should change IV e to read graph and indicate all significant points.

also add red pts to the foto.

93

	2	5	T
100	0	0	0
90	5	5	
80	6	2	
70	6	8	
60	2	4	
59	2	1	

99

1664/21 = 79.2

1555/20 = 77.75

52

3219/41 = 78.51

95

	5	6	T
100	0	1	1
90	5	5	10
80	5	2	12
70	2	1	3
60	2	0	2
59	0	2	2

100

45

1177/14 = 84.07

923/11 = 83.91

2100/25 = 84.0

Honors Analysis Name ANL Ch3 1993

Copy original problem. Per _____ Date 1995

Convince *me* that you understand the concept! 2002

Chapter 3 Exam

I Given $f(x)=\begin{cases}\frac{2}{3}x+1 & -3<x<-1\\ 3 & x=-1\\ -x-2 & -1<x<1\\ x-4 & 1<x<3\\ -\frac{4}{3}x+3 & 3\le x<5\end{cases}$ (20 pts tot)

a) Prove $f(x)$ is or is not continuous at $x=-1$

b) Prove $f(x)$ is or is not continuous at $x=1$

c) Prove $f(x)$ is or is not continuous at $x=3$

II *Using the definition of the derivative* which yields a function,

find $f'(x)$ given $f(x)=x^2+1$. (15 pts)

use the 1st Def of the derivative

III Given $f(x)=|x^2-4|$ *Prove* $f'(2)$ does not exist. (15 pts)

IV Given $f(x)=4x^3-12x^2+8$ (35 pts tot)

a) State the coordinates of all x-intercepts of $f(x)$.

b) State the intervals (using proper notation) where $f(x)$ is increasing; decreasing.

c) Identify and state the coordinates of all extrema. **Justify.**

d) State the coordinates of any Points of Inflection. **Justify.**

e) Draw an accurate graph of $f(x)$. Specifically show all coordinates found above.

V Find the following limits. (If they do not exist, explain fully how you know that.) (5 pts ea)

a) $\lim\limits_{x\to 1}\dfrac{x+1}{x^2-1}$ b) $\lim\limits_{x\to 2}\dfrac{x^3-8}{x-2}$ c) $\lim\limits_{x\to 3}\dfrac{\sqrt{x-1}}{\sqrt{x-5}}$

Extra Credit ---------- 5 pts ----------

A square of perimeter 20 is inscribed in a square of perimeter 28. What is the greatest distance between a vertex of the inner square and a vertex of the outer square?

I $f(x) = \begin{cases} kx^2 & \text{if } x \le 2 \\ 2x+k & \text{if } x > 2 \end{cases}$

a) $f(2) = 4k$ ① $\to 4 \cdot \frac{4}{8} = \frac{16}{3}$ ③

$\lim_{x\to 2^-} kx^2 \overset{\text{plug in}}{=} 4k$ ③ $=$ $\lim_{x\to 2^+} 2x+k \overset{\text{plug in}}{=} 4+k$

$3k = 4$ ②

$k = \frac{4}{3}$

$\therefore \lim_{x\to 2} f(x) = \frac{16}{3}$

$\begin{cases} \frac{4}{8}x^2 & \text{if } x \le 2 \\ 2x+\frac{4}{3} & \text{if } x > 2 \end{cases}$

b) $\lim_{x\to 2^-} \frac{\frac{4}{8}x^2 - \frac{16}{3}}{x-2}$

$\lim_{x\to 2^-} \frac{\frac{1}{3}(x^2-4)}{x-2}$

$\lim_{x\to 2^-} \frac{\frac{1}{3}(x-2)(x+2)}{x-2}$

$\lim_{x\to 2^-} \frac{1}{3}(x+2) = \frac{16}{3}$ ←

$\lim_{x\to 2^+} \frac{2x+\frac{4}{3} - \frac{16}{3}}{x-2}$

$\lim_{x\to 2^+} \frac{2x-4}{x-2}$

$\lim_{x\to 2^+} \frac{2(x-2)}{(x-2)} = 2$

∴ limit DNE since LHL ≠ RHL

II $f(x) = \frac{x^2-1}{x}$

$\lim_{h\to 0} \left(\frac{\frac{(x+h)^2-1}{x+h} - \frac{x^2-1}{x}}{h} \right)$

$\lim_{h\to 0} \left(\frac{\frac{x^2+2hx+h^2-1}{x+h} - \frac{x^2-1}{x}}{h} \right)$

$\lim_{h\to 0} \left(\frac{x^3+2hx^2+xh^2-x-(x^3+hx^2-x-h)}{hx(x+h)} \right)$

$\lim_{h\to 0} \left(\frac{x^3+2hx^2+xh^2-x-x^3-hx^2+x+h}{hx(x+h)} \right)$

$\lim_{h\to 0} \left(\frac{hx^2+xh^2+h}{hx(x+h)} \right)$

$\lim_{h\to 0} \frac{h(x^2+xh+1)}{hx(x+h)}$

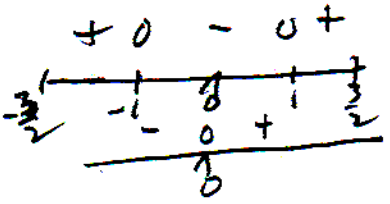
$\lim_{h\to 0} \frac{x^2+xh+1}{x(x+h)} = \frac{x^2+1}{x^2}$

III

$f(x) = x^3 - 3x + 2$

$f'(x) = 3x^2 - 3$

$f''(x) = 6x$



	1	0	-3	2
$-\frac{3}{2}$	1	$-\frac{3}{2}$	$-\frac{3}{4}$	$\frac{25}{8}$
-1	1	-1	-2	4
2				2
1	1	1	-2	0
$\frac{3}{2}$	1	$\frac{3}{2}$	$-\frac{3}{4}$	$\frac{7}{8}$

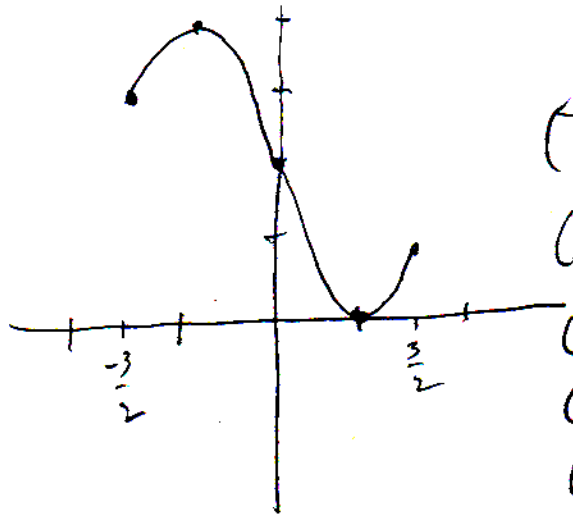
$(x-1)(x^2+x-2)$

$(x+2)(x-1)$

$(x-1)^2(x+2)$

x intercepts $-2, 1$

III



$(\frac{3}{2}, \frac{25}{8})$ min END PT inc function (Rel)

$(-1, 4]$ max of this (absolute)

$(0, 2)$ PI down to up

$(1, 0)$ min of this (abs)

$(\frac{3}{2}, \frac{7}{8})$ max END PT inc to it. (Rel)

IV

a) $\lim_{x \to 4} \left(\frac{1}{x} - \frac{1}{4}\right)$

$\frac{1}{4} - \frac{1}{4}$

0

b) $\lim_{x \to 4} \left[\left(\frac{1}{x} - \frac{1}{4}\right)\left(\frac{1}{x-4}\right)\right]$

$\lim_{x \to 4} \left[\frac{4-x}{4x} \cdot \frac{1}{x-4}\right]$

$\lim_{x \to 4} \left[\frac{-1}{4x}\right]$

$-\frac{1}{16}$

c) $\lim_{x \to 4} \left[\left(\frac{1}{x} - \frac{1}{4}\right)(x-4)\right]$

$0 \ (4-4)$

0

d) $\lim_{x \to 4} \left[\left(\frac{1}{x} - \frac{1}{4}\right)\left(\frac{1}{x-4}\right)^2\right]$

$\lim_{x \to 4} \left[\frac{4-x}{4x} \cdot \frac{1}{(x-4)^2}\right]$

$\lim_{x \to 4} \left(\frac{-1}{4x(x-4)}\right)$

$\lim_{x \to 4^-} \frac{-1}{4x(x-4)} \to +\infty$

$\therefore$ DNE.

Honors Analysis

Name AnL Ch 3

Per _____ Date 2003

Copy original problem.

Convince *me* that **you** understand the concept!

No Calculators, of course.

Make up.

Chapter 3 Exam

I Given $f(x) = \begin{cases} kx^2 & \text{if } x \le 2 \\ 2x + k & \text{if } x > 2 \end{cases}$ (30 pts tot)

a) Determine k so that $f(x)$ continuous.

b) Prove $f'(2)$ does not exist.

II *Using the definition of the derivative* which yields a function,

find $f'(x)$ given $f(x) = \dfrac{x^2 - 1}{x}$. (15 pts)

III Given $f(x) = x^3 - 3x + 2$; $x \in \left[-\frac{3}{2}, \frac{3}{2}\right]$ (40 pts tot)

a) Identify and state the coordinates of all extrema and axis intercepts. **Justify**.

b) Make an $f''(x)$ numberline with proper placement of pluses and minus.
State the coordinates of any Points of Inflection.

c) Draw an accurate graph of $f(x)$. Specifically show all coordinates found above.

IV Find the following limits. (If they do not exist, explain fully how you know that.) (15 pts tot)

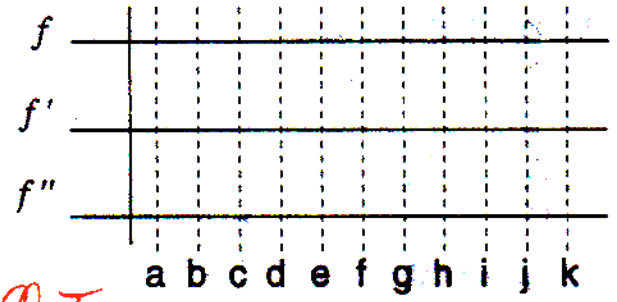
a) $\lim\limits_{x \to 4} \left(\frac{1}{x} - \frac{1}{4}\right)$

b) $\lim\limits_{x \to 4} \left[\left(\frac{1}{x} - \frac{1}{4}\right)\left(\frac{1}{x-4}\right)\right]$

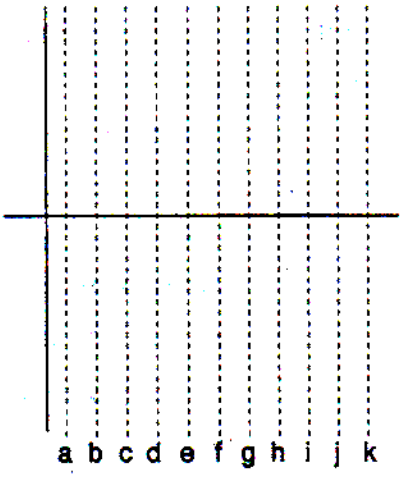
c) $\lim\limits_{x \to 4} \left[\left(\frac{1}{x} - \frac{1}{4}\right)(x-2)\right]$

d) $\lim\limits_{x \to 4} \left[\left(\frac{1}{x} - \frac{1}{4}\right)\left(\frac{1}{x-4}\right)^2\right]$

Ia



Ib

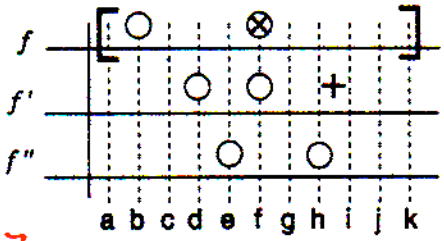


an excellent, relatively easy, EXAM.
a few nonextenders dropped - mostly but a few still remain.

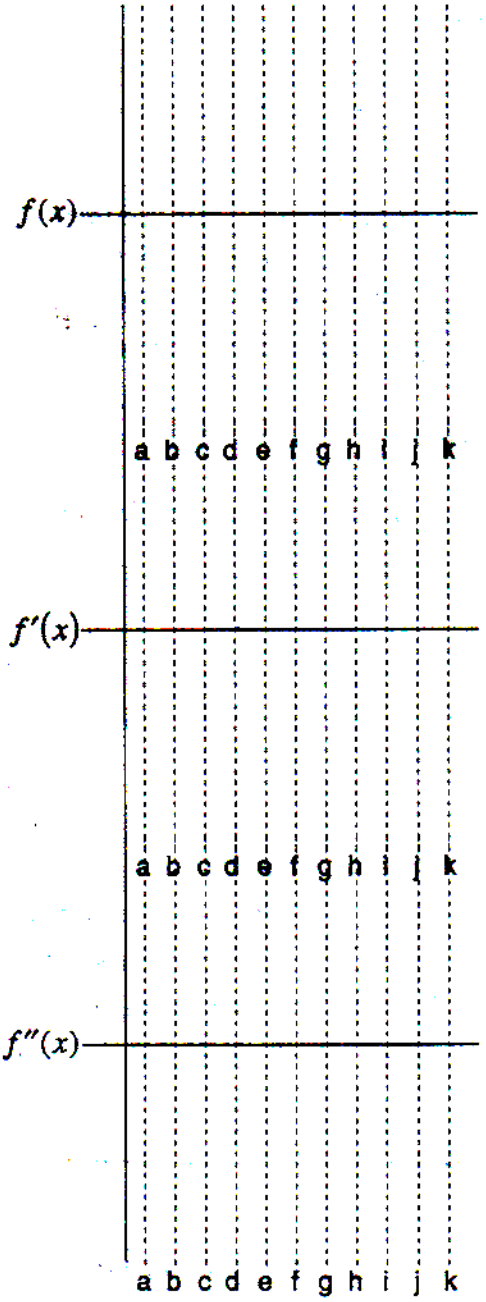
II

Below is the same graphic which is on the other side of this paper. It is reproduced here for your convenience. You are to fill in the graphic with pluses and minuses, etc. and then use the number lines to sketch the graph of the appropriate function on the proper axis.

Put the rest of your answers on the newsprint as usual.



2003



	2	3	
100	1	1	2
90	3	3	6
80	6	3	9
70	5	8	13
60	0	1	1
59	2	3	5

104

1338/17 → 1446/19 → 723 → 35 → 761

Honors Analysis Name _____

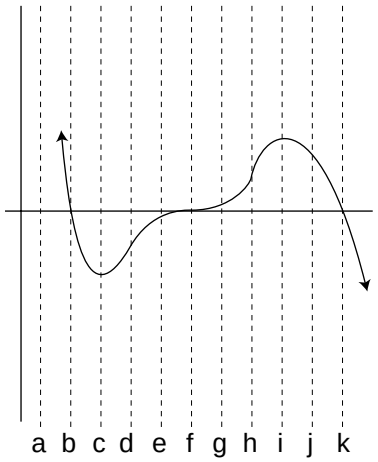
Copy original problem. Per _____ Date _____

Convince *me* that **you** understand the concept***!***

No Calculators.

Chapter 3 Exam

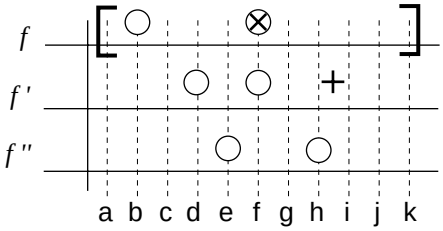
I Given the graph on the right: (20 pts tot)



a) Assume the given graph is $f(x)$.
Properly fill in the $f(x)$, $f'(x)$, $f''(x)$ ***numberlines on the back of this page.***.

b) Assume the given graph is $f'(x)$. **On the axis on the back of this paper** sketch $f(x)$. Specifically state coordinates of all extrema and PI. (Typically you might have a response such as: "$(a, f(a))$ f' test".) Justify, of course.

II Given:



(30 pts tot)

a) Use the single *y*–axis and three *x*–axes **on the back of this page** to graph f, f', and f''.

b) Specifically state the coordinates of all extrema and points of inflection on f. Justify, of course. (As in Ib, above. Be sure to properly state coordinates.)

c) Specifically identify extrema on $f'(x)$. Justify. (Notice this asks about extrema in $f'(x)$)

III Given $f(x) = \dfrac{x}{x-1}$ for $-3 \le x \le 3$. (25 pts tot)

a) Using the definition of the derivative which yields a function, find $f'(x)$.

b) Using your answer to part a, find $f'\left(\frac{3}{2}\right)$

IV Given $f(x) = \begin{cases} |x+2| & \text{if } -\frac{5}{2} \le x < -1 \\ 2 - x^2 & \text{if } -1 \le x < 1 \\ 3 - 2x & \text{if } 1 \le x \le \frac{5}{2} \end{cases}$ (25 pts tot)

a) **Prove** f is continuous at $x = -1$

b) **Prove** f is continuous at $x = 1$

c) **Prove** $f'(-1)$ does or does not exist.

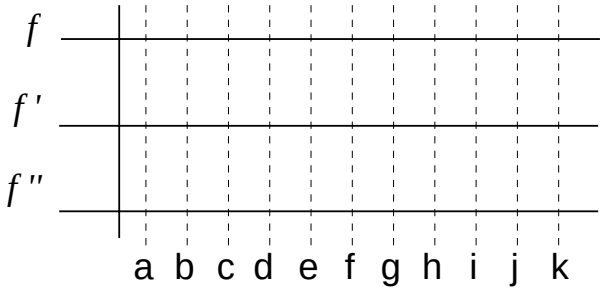
b) **Prove** $f'(1)$ does or does not exist.

e) **Prove** $f'(-2)$ does or does not exist.

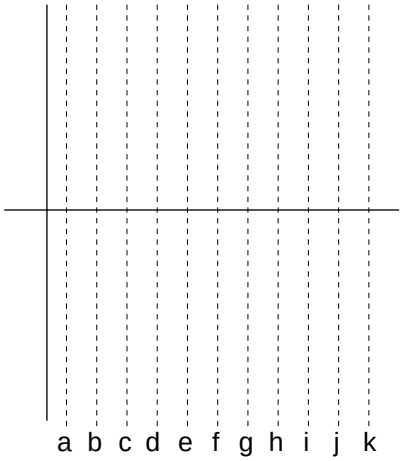
Extra Credit ---------- 5 pts ----------

Draw the graph of $f(x)$ from section IV above.

I a



I b



II

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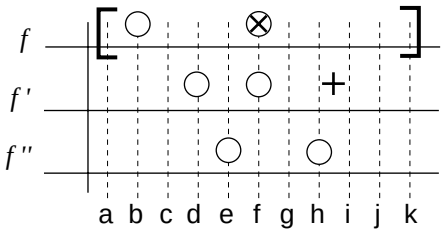
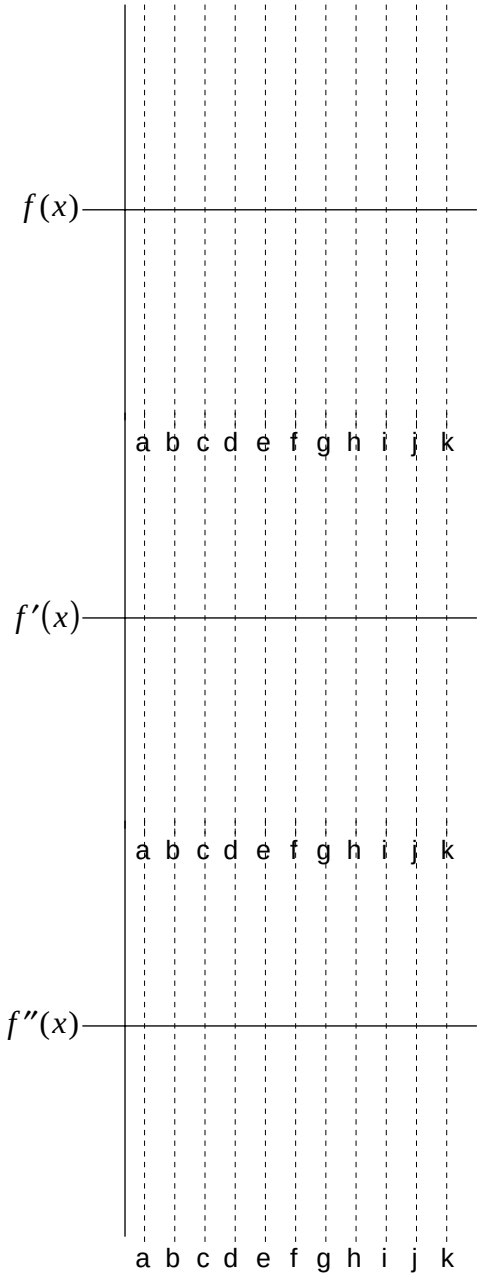
Put the rest of your answers on the newsprint as usual.





Honors Analysis Chapter 3

Don't even think about getting 30 minutes behind!

Exam Chapter 2

• • • • • • • • • • • • • • • •

1 R/C Pg 109-113

• • • • • • • • • • • • • • • •

2 Write out Q & A to Oral Ex pg 113 1 – 11

3/ 7

Important Lecture
3 unLIMITed Enjoyment section I

3/ 10

4, #5 unLIMITed Enjoyment section II, III

3/ 11

Important Lecture
6 Pg 119 1 - 10, also graph pblms numbered 5 - 9 Pg 125 13 - 24. Use abscissa = 2 (That means the x value is two.)

3/ 12

7 Pg 125 Replace text instructions with: Find $f'(x)$ using 2nd def on problems 1 - 12. **Also** for each of 1 - 12, find the x–value where $f'(x) = 0$

3/13

8 Pg 130 1 –18 (1,7,13; 2,8,14; etc.) Also
9 Page 136
1, 2, 4, 6

3/ 14

10,11 F, F ', F "
Finish the handout

3/ 17

#12,13 Number Lines-R-Us

3/18

Begin
#14, 15, 16, 17, 18
"Just a Few Interesting Problems."

3/19

Minimum Day
Enjoying

"Just a few ..."

3/20

Minimum Day
Turn in "Just a Few ...

• • • • • • • • • • • • • • • •

19 "Find the Pretender"

• • • • • • • • • • • • • • • •

Begin **"Mr. Summary"**
20, 21, 22, 23, 24

3/ 21

Turn in #19

3/ 24

Industrial Strength
el Quizzo
value: 4 assignments!

3 / 25

3/26

Put last year's exam on the chalk boards

3/27

EXAM CHAPTER III

3/ 28

$(x^3 \pm y^3) =$
$(x \pm y)(x^2 \mp xy + y^2)$

$(x \pm y)^3 =$
$(x^3 \pm 3x^2y \mp 3xy^2 \pm y^3)$

2003

You didn't do this last year!
This chapter is the backbone of the course. You *must* make every effort to understand *completely*.
Master this stuff!

I

4) $(x+1)^6$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$x^6+6x^5+15x^4+20x^3+15x^2+6x+1$ 5 pts

or -2

3) $f(x) = 3x^4-8x^3+6x^2+1$

$f'(x) = 12x^3-24x^2+12x$

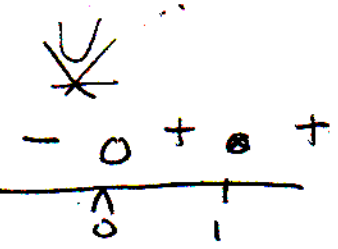
$12x(x^2-2x+1)$

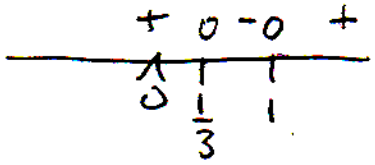
$12x(x-1)^2$

$f''(x) = 36x^2-48x+12$

$12(3x^2-4x+1)$

$12(3x-1)(x-1)$





	3	−8	6	0	1
1/3	3	−7	11/3	11/9	38/27
1	3	−5	1	1	2

10 pts

minimum at (0, 1)

f'' test

LAZY PI at (1, 2)

3 pts Active PI at $\left(\frac{1}{3}, \frac{38}{27}\right)$

total 10 pts

III $f(x) = \begin{cases} x^2 & x \le 1 \\ 2x-1 & x>1 \end{cases}$

A) $\lim_{x\to 1} f(x) = f(1)$ $f(1)=1$

$\lim_{x\to 1^-} f(x) = 1^2 = 1$ } $\lim_{x\to 1} f(x) = 1$

$\lim_{x\to 1^+} f(x) = 2-1 = 1$

$f(1) = \lim_{x\to 1} f(x)$ so f is cont.

25

-3

B) $\lim_{x\to 1^-} \frac{x^2-1}{x-1} = \lim_{x\to 1} x+1 = 2$

$\lim_{x\to 1^+} \frac{2x-1-1}{x-1} = \lim_{x\to 1^+} \frac{2x-2}{x-1} = 2$

$\therefore \lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = 2$

$\therefore f'(1)$ exists and $=2$

No Proof -10

C) $f'(x) = ?$

① $f(x) = (4x^3-5x)^2$

$16x^6-40x^4+25x^2$

f' $96x^5-160x^3+50x$

② $f(x) = x^{\pi}+3$

$f'(x) = \pi x^{\pi-1}$

③ $f(x) = 4^2-1$

$f'(x) = 0$

II $f(x) = (x-3)^2 = x^2-6x+9$

$\lim_{h\to 0} \frac{(x+h)^2-6(x+h)+9-x^2+6x-9}{h}$

$\lim_{h\to 0} \frac{x^2+2xh+h^2-6x-6h+9-x^2+6x-9}{h}$

$\lim_{h\to 0} \frac{2xh+h^2-6h}{h} = \lim_{h\to 0} 2x-6+h =$

$2x-6$

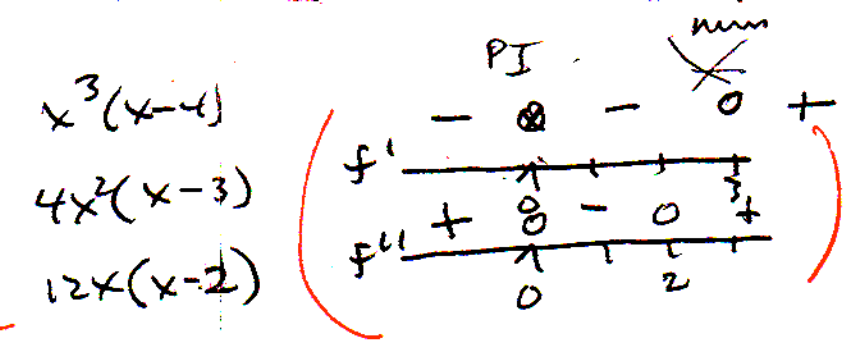
10 pts

IV

$f(x) = x^4 - 4x^3 = x^3(x-4)$

$f'(x) = 4x^3 - 12x^2 \quad 4x^2(x-3)$

$f''(x) = 12x^2 - 24x \quad 12x(x-2)$

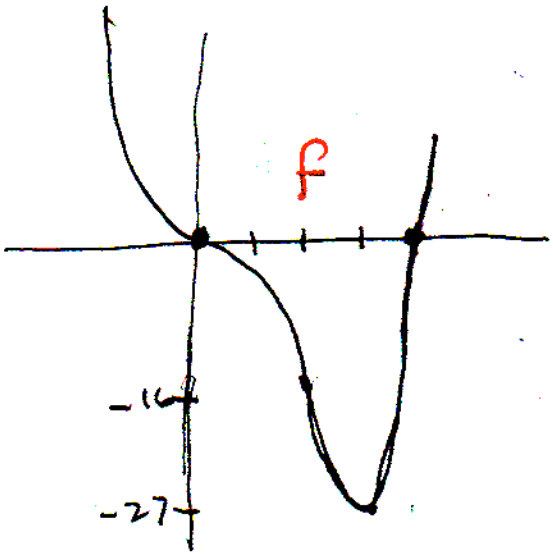


$f(0) = 0 \quad (0,0)$

$f(3) = 3^3(3-4) = -27 \quad (3,-27)$

$f(2) = 8(-2) = -16$

total 3 pts



B (0,0) PI Lazy f' test

(3,-27) Abs min f'' test

C (2,-16) Active PI (not critical value)

D f concave up $\forall x < 0$ or $\forall x > 2$

E f decreasing $\forall x < 3, x \neq 0$

V

4) $\lim_{x\to 2} \frac{\sqrt{x^2+5}-3}{x-2} \cdot \frac{\sqrt{\quad}+3}{\sqrt{\quad}+3}$

$\lim_{x\to 2} \frac{x^2+5-9}{(x-2)(\sqrt{\quad}+3)} = \lim_{x\to 2} \frac{x+2}{\sqrt{x^2+5}+3} \quad \frac{4}{6} = \frac{2}{3}$

5 pts

eq

3) $\lim_{x\to 0} \sqrt{1+\frac{1}{x}} - \sqrt{\frac{1}{x}}$

$\lim_{x\to 0} \frac{\sqrt{x+1}-1}{\sqrt{x}} \cdot \frac{\sqrt{\quad}+1}{\sqrt{\quad}+1} = \lim_{x\to 0} \frac{x+1-1}{\sqrt{x}(\sqrt{x+1}+1)} =$

$\lim_{x\to 0} \frac{x}{\sqrt{x}(\sqrt{x+1}-1)} = \lim_{x\to 0} \frac{\sqrt{x}}{\sqrt{x+1}+1} = \frac{0}{2} = 0$

c) $\lim_{x\to 1} \left[\frac{1}{1-x^3} + \frac{1}{3(x-1)}\right] = \lim_{x\to 1} \left[\frac{3-(1+x+x^2)}{3(1-x)(1+x+x^2)}\right]$

$\lim_{x\to 1} \left[\frac{x^2+x-2}{3(x-1)(1+x+x^2)}\right] = \lim_{x\to 1} \left[\frac{(x+2)(x-1)}{3(x-1)(1+x+x^2)}\right]$

$\lim_{x\to 1} \left[\frac{x+2}{3(1+x+x^2)}\right] = \frac{3}{3\cdot 3} = \frac{1}{3}$

Extra.

$f(x) = ax^3 + bx^2$

$f(1) = a + b = 2$

$f'(x) = 3ax^2 + 2bx$

$f''(x) = 6ax + 2b$

$f''(1) = 6a + 2b \overset{\text{set}}{=} 0$

$3a + b = 0$

$-a - b = -2$

$2a = -2$

$a = -1$

$-1 + b = 2$

$b = 3$

$f(x) = -x^3 + 3x^2$

ie $a = -1$

$b = 3$

Put deleted pblms on Sept Quiz.

Also on Quiz give f', f'' + lims to use as "facts"

Listed items needs to be deleted because these students haven't yet learned to budget their exam time. They put too much effort into I B.

	2	5	T
100	0	0	0
90	1	2	3
80	6	3	9
70	7	9	16
60	6	3	9
54	4	6	10
	71,3	69,5	70,35

98

↓

0 ← 2 cheaters

14 Games. (a repeat Analysis student!)

ANALYSIS Chapter 3 EXAM Name ANALYSIS

Convince ME that YOU know the concept !! ch 3 Per 3/27/87

I Warm up problems

A) Given $(X+1)^6$. Clear the parentheses (5 pts)

delete B) Given $F(X) = 3X^4 - 8X^3 + 6X^2 + 1$. Determine the coordinates of all extrema and P.I. JUSTIFY. (Don't forget to draw clearly labeled numberlines.) (10 pts)

C) Find $F'(X)$, given $F(X)$ = : 5 pts ea. (~~total 10 pts~~)

(1) $(4X^3 - 5X)^2$ (2) $X^{\pi} + 3$ (3) $4^2 - 1$

II Given : $F(X) = (X-3)^2$. Using the "definition of the derivative" which yields a function, find $F'(X)$. 15 (~~10 pts~~)

III Given $F(X) = \begin{cases} X^2 & ; X \leq 1 \\ 2X - 1 & ; X > 1 \end{cases}$ ~~total 25~~ 15 pts ea.

A) **PROVE** F is continuous at X = 1.

B) **PROVE** $F'(1)$ DNE.

IV Given : $F(X) = X^4 - 4X^3$ 30 (~~total 30 pts~~)

A) Find F' and F''. Draw appropriate "number line" graphs.

B) Find the COORDINATES of the critical points of F. Identify them as Max, Min, or PI. **JUSTIFY** your responses.

C) Determine the COORDINATES of any PI which are NOT critical values.

D) State the intervals where F is concave UP.

E) State the intervals where F is decreasing.

F) Sketch the graph of F.

V Determine the following limits (5 pts ea)

4) $\lim_{x \to 2} \dfrac{\sqrt{x^2+5} - 3}{x-2}$ B) $\lim_{x \to 0} \sqrt{1 + \frac{1}{x}} - \sqrt{\frac{1}{x}}$ delete C) $\lim_{x \to 1} \left[\dfrac{1}{1-x^3} + \dfrac{1}{3(x-1)} \right]$

EXTRA CREDIT ----------

Given $F(X) = aX^3 + bX^2$.

Determine a and b so the graph of F will have a point of inflection at (1, 2).

I

4) Explain "The slope of the tangent to the ... of the ..."

B) $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

c) explain $\lim_{x\to a} f(x) = L$ must include $x\to a^+$, $x\to a^-$

L is a #.

II

4) explain "pencil does not leave the paper" version of cont

B) $\lim_{x\to a} f(x) = f(a)$ agn $x\to a^+$, $x\to a^-$, $a \in D_f$

c) $\lim_{x\to 3} \frac{\sqrt{2x+3}-3}{x-3} \cdot \frac{\sqrt{\ }+3}{\sqrt{\ }+3} = \lim_{x\to 3} \frac{2x+3-9}{(x-3)\sqrt{2x+3}+3} = \lim_{x\to 2} \frac{2(x-3)}{(x-3)(\sqrt{2x+3}+3}$

$\lim_{x\to 3} \frac{2}{\sqrt{2x+3}+3} = \frac{2}{3+3} = \frac{1}{3}$

to follow def of continuity, The limit exists and is $\frac{1}{3}$ $\therefore$ $f(a) = \frac{1}{3}$ $\therefore$ $z = \frac{1}{3}$ and f is continuous.

III $f(x) = x + \frac{1}{x}$

$\lim_{h\to 0} \frac{x+h+\frac{1}{x+h} - x - \frac{1}{x}}{h} = \lim_{h\to 0} \frac{x^2h + xh^2 + x - x - h}{xh(x+h)} =$

$\lim_{h\to 0} \frac{x^2+xh-1}{x(x+h)} = \frac{x^2-1}{x^2}$

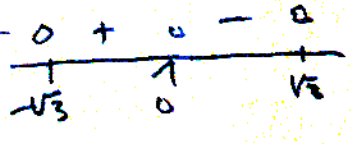
IV $[-2, 2]$

$f(x) = x^4 - 6x^2$

$x^2(x^2 - 6)$

$f'(x) = 4x^3 - 12x$

$4x(x^2 - 3)$



$f''(x) = 12x^2 - 12$

$12(x^2 - 1)$

$f(\pm\sqrt{3}) = 9 - 18 = -9$

$f(0) = 0$

coord of critical pts:

$(\sqrt{3}, -9)$ $(0, 0)$ $(\sqrt{3}, -9)$

$f(\pm 2) = 16 - 24 = -8$

$(-2, -8)$ rel max decreasing to it

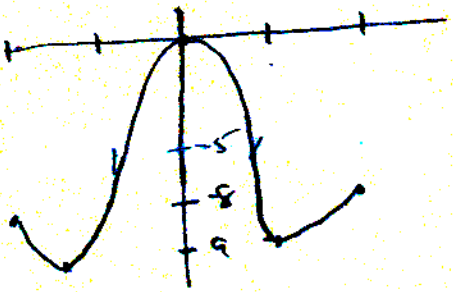
$(-\sqrt{3}, -9)$ abs min f' cont

$(0, 0)$ Abs max f' cont

$(\sqrt{3}, -9)$ abs min f' cont

$(2, -8)$ rel max increasing to it

f concave up $|x| > 1$



V

4) $\lim_{x \to 1} \frac{\frac{1}{x} - 1}{x - 1}$

$\lim_{x \to 1} \frac{1 - x}{x(x-1)} = -1$

13) $\lim_{x \to 0} \frac{|x| - x}{|x|}$

$\lim_{x \to 0^-} \frac{-x - x}{-x} = -2$

$\lim_{x \to 0^+} \frac{x - x}{x} = 0$

$-2 \neq 0$ ∴ limit DNE.

c) $\lim_{x \to 0} \left[\frac{3 + 2x}{x + 5x^2} - \frac{3}{x} \right] = \lim_{x \to 0} \left[\frac{3 + 2x - 3 - 15x}{x(1 + 5x)} \right]$

$\lim_{x \to 0} \frac{-13}{1 + 5x} = -13.$

Extra

$f(x) = |x - 2|$

must show $\lim_{x \to 2} f(x) = f(2)$ AND $\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$ DNE

and conclude ① ≠ ②

so limit DNE, since limit DNE, then der DNE also.

	S	S	T
100	0	1	1
90	6	3	10
80	3	1	3
70	3	7	10
60	2	4	6
54	8	5	13
	71.64	70.71	71.19

101

32

Don't give all the "notes" handouts. They simply don't read them for understanding. Instead, the handouts however.

ANALYSIS Chapter 3 EXAM Name Ch 3 C 3/488

Per ANALYSIS 8 MW

3/28/88

I EXPLAIN 'UMS (25 pts)

A) When we have found the derivative of a function at a specific point, what EXACTLY have we found? Explain fully. Do NOT use examples.

B) There are two "definitions of the derivative". Write both.

C) Translate the following to English and explain the meaning --- be complete.

$$\lim_{x \to a} F(x) = L$$

II Continuity (20 pts)

A) Explain the concept of continuity in layman's terms.

B) What is the mathematical definition of continuity?

C) What choice of Z, if any, would you make to make F continuous at x = 3 given: (explain yourself)

$$F(x) = \begin{cases} \dfrac{\sqrt{2x+3} - 3}{x - 3} & x \neq 3 \\ Z & x = 3 \end{cases}$$

III Using the definition of the derivative ($h \to 0$ method) find F′ given $F(x) = x + \frac{1}{x}$ (10 pts)

IV Given $F(x) = x^4 - 6x^2$ AND D_F $x \in [-2, 2]$ (30 pts)

A) Find F′ and F″
B) Determine the coordinates of all critical points.
C) Identify all extrema and justify your conclusions.
D) Specifically state the interval(s) where F is concave UP.
E) Sketch a graph of F(x) based on your conclusions.

V Find the following limits, if they exist. If they do not exist, explain how know that. (5 pts ea)

A) $\lim_{x \to 1} \dfrac{\frac{1}{x} - 1}{x - 1}$ B) $\lim_{x \to 0} \dfrac{|x| - x}{|x|}$ C) $\lim_{x \to 0} \left[\dfrac{3 + 2x}{x + 5x^2} - \dfrac{3}{x} \right]$

EXTRA CREDIT ---------- 5 PTS ----

Given $F(X) = |X - 2|$, PROVE F is continuous at $X = 2$ AND $F'(2)$ DNE.

I $f(x) = |x-2|$

A) $\lim_{x\to 2} f(x) \overset{?}{=} f(2)$

$\lim_{x\to 2^-} -(x-2) = 0$ ⟵ $=$ ⟶ $\lim_{x\to 2^+} x-2 = 0$ ⟶ $\therefore \lim_{x\to 2} f(x) = 0$

$f(2) = |2-2| = 0$ ⟵ $=$

$\therefore$ f continuous at $x=2$.

B) $\lim_{x\to 2} \frac{|x-2| - |2-2|}{x-2}$

$\lim_{x\to 2^-} \frac{-(x-2)}{x-2} = -1$ ⟵ $\neq$ ⟶ $\lim_{x\to 2^+} \frac{x-2}{x-2} = 1$

$\therefore$ limit DNE. But since limit IS Def of Der, Der DNE!

II $f(x) = x - \frac{1}{x}$

$$\lim_{h\to 0} \frac{x+h - \frac{1}{x+h} - x + \frac{1}{x}}{h}$$

$$\lim_{h\to 0} \frac{1}{h}\left(\frac{x(x+h)h - x + x+h}{x(x+h)}\right]$$

$$\lim_{h\to 0} \frac{1}{h}\left(\frac{x^2h + xh^2 + h}{x(x+h)}\right]$$

$$\lim_{h\to 0} \frac{x^2 + xh + 1}{x(x+h)} = \frac{x^2+1}{x^2}$$

III $f(x) = x^3 - 2x^2 - 2x$

	1	−2	−2	0
1		1	−1	−3	−3
1		1	0	−3	

$y + 3 = -3(x-1)$

$y = -3x$

IV $f(x) = 3x^5 - 5x^3$ $\quad D_f : x \in [-2, 2]$

$x^3(3x^2 - 5)$

$f'(x) = 15x^4 - 15x^2$

$15x^2(x^2-1)$

$f''(x) = 60x^3 - 30x$

$30x(2x^2-1)$

f' : + 0 − 0 − 0 + (B)

f'' : − 0 + 0 − 0 +

A) critical values of $x \in \{0, \pm 1\}$

C) $f(-2) = -56$ $(-2, -56)$ Abs min INC to it

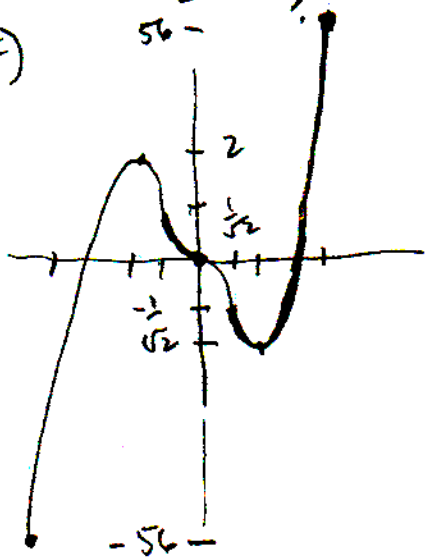
$f(-1) = +2$ $(1, -2)$ Rel max f' test

$f(+1) = -2$ $(-1, 2)$ Rel min f' test

$f(2) = 56$ $(2, 56)$ Abs max INC to it

D) f concave up $\left(\frac{-1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{2}}, 2\right]$

E)



$-f(12-5)$

$8(7)$

V

A) $\lim_{x\to 2^-} \frac{4-x^2}{\sqrt{2+x-x^2}}$ $\sqrt{}$ $\sqrt{}$

$\lim_{x\to 2^-} \frac{4-x^2\sqrt{2+x-x^2}}{2+x-x^2}$

$\lim_{x\to 2^-} \frac{(2-x)(2+x)\sqrt{2+x-x^2}}{(2+x)(1-x)}$

$\frac{0\cdot 0}{-1} = 0$

B) $\lim_{x\to 9} \frac{6-2\sqrt{x}}{x-9}$

$\lim_{x\to 9} \frac{2(3-\sqrt{x})}{(\sqrt{x}-3)(\sqrt{x}+3)}$

$\lim_{x\to 9} \frac{-2}{\sqrt{x}+3} = -\frac{1}{3}$

C) $\lim_{x\to 0} \frac{|x|-x}{x}$

$\lim_{x\to 0^-} \frac{-2x}{x} = -2$

$\lim_{x\to 0^+} \frac{x-x}{x} = 0$

① ≠ ② ∴ $\lim_{x\to 0} \frac{|x|-x}{x}$ DNE

D) $\lim_{x\to 1} \frac{x^2-1}{\sqrt{x}-1}\frac{(\sqrt{x}+1)}{(\sqrt{x}+1)}$

$\lim_{x\to 1} \frac{(x^2-1)(\sqrt{x}+1)}{x-1}$

$\lim_{x\to 1} (x+1)(\sqrt{x}+1)$

$2\cdot 2 = 4$

E) $\lim_{x\to 4^+} 2^3+1 = 9$

F) $\lim_{x\to -2} \frac{\frac{1}{\sqrt{x^2}} - \frac{1}{2}}{x+2}$

$\lim_{x\to -2} \frac{\frac{1}{|x|} - \frac{1}{2}}{x+2}$

$\lim_{x\to -2} \frac{-\frac{1}{x} - \frac{1}{2}}{x+2}$

$\lim_{x\to -2} \frac{-\left(\frac{2+x}{2x}\right)}{x+2}$

$\lim_{x\to -2} -\frac{1}{2x} = +\frac{1}{4}$

EXTRA

$$f(x) = \begin{cases} x^2 & x \le 1 \\ 2x-1 & x > 1 \end{cases}$$

A) $\lim_{x\to a} f(x) = f(a)$

$f(1) = 1^2 = 1$ ←

$\lim_{x\to 1^-} x^2 = 1$ ← =

$\lim_{x\to 1^+} 2x-1 = 2-1 = 1$ ← =

$\therefore \lim_{x\to 1} f(x) = 1$ ←

$\therefore$ f is continuous at $x=1$

B) $\lim_{x\to 1} \frac{f(x)-f(1)}{x-1}$

$\lim_{x\to 1^-} \frac{x^2-1}{x-1}$

$\lim_{x\to 1^-} x+1 = 2$ ← =

$\lim_{x\to 1^+} \frac{2x-1-1}{x-1}$ = ∴ Derivative DOES EXIST!

$\lim_{x\to 1^+} \frac{2x-2}{x-1} = 2$ ←

9/12/85

	5	6	T
100↑	1	0	1
90↑	2	6	8
80	9	6	15
70	12	5	17
60	6	1	7
59	1	2	3
	77.5	82.7	79.53

100 ↓ 26

Excellent exam

Absolutely no changes.

89

	2	5	T
100↑	1	0	1
90↑	4	4	8
80↑	2	3	5
70	2	3	5
60	6	4	10
59	5	2	17
	69.9	73.7	71.6

Estimation hunt
average
so did Band trip
to Hawaii!

Honors ANALYSIS NAME ch 3

Convince ME that *YOU* know the concept! PER _____ DATE 4/7/89

Chapter 3 EXAM

I Given $F(x) = |x-2|$ (10 pts ea)

A) *PROVE* F(x) is continuous at x = 2.

B) *PROVE* F'(x) DNE at x = 2

II Using the definition of the derivative which yields a function, find F'(x) given

$F(x) = x - \frac{1}{x}$ (10 pts)

III Given $F(x) = x^3 - 2x^2 - 2x$ Find the equation of the tangent line to F(x) at the point (1 , F(1)). (10 pts)

IV Given F(x) = $3x^5 - 5x^3$ and D_F $x \in [-2, 2]$ (Total 30 pts)

A) Determine the critical values of x .
B) Set up number lines for F ' and F " — label properly.
C) Identify the coordinates of all extremum (do NOT include ANY points of inflection.) (Be very sure your reasonning is clear to ME.)
D) Determine and specifically list the intervals where F (x) is concave UP .
E) Draw and properly label a graph of F (x) .

V Determine the following limits (if they exist). If a limit does NOT exist, explain how you know this. (5 pts ea)

A) $\lim_{x \to 2^-} \frac{4 - x^2}{\sqrt{2 + x - x^2}}$

B) $\lim_{x \to 9} \frac{6 - 2\sqrt{x}}{x - 9}$

C) $\lim_{x \to 0} \frac{|x| - x}{x}$

D) $\lim_{x \to 1} \frac{x^2 - 1}{\sqrt{x} - 1}$

E) $\lim_{x \to 4^+} 2^3 + 1$

F) $\lim_{x \to -2} \frac{\frac{1}{\sqrt{x^2}} - \frac{1}{2}}{x + 2}$

EXTRA CREDIT ---------- 5 points

Given $F(x) = \begin{cases} x^2 & ; x \le 1 \\ 2x - 1 & ; x > 1 \end{cases}$

A) *PROVE* F (x) is continuous at x = 1 .

B) *PROVE* F ' (x) DNE at x = 1

I A) When you have found the derivative of a fct'n at a pt, you have found the slope of the tangent line to the fct'n at that pt.

B) $\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = f'(a)$ $\quad f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

same: Both are limits of slopes $(x, f(x))$, $(a, f(a))$ and $(x, f(x))$ and $(x+h, f(x+h))$

diff: The 1st one yields a number - the slope at pt $(a, f(a))$ the second yields a fct'n which is the slope at any pt $(x, f(x))$

C) $\lim_{x\to a} f(x) = f(a)$ is the def of continuity

the right side, $f(a)$ establishes that the pt $(a, f(a))$ Exists. The left side implies that the limit from the left = limit from above thus, the limit exists & also if it is = to $f(a)$, the fct'n is cont at that point.

II $f(x) = (x+3)^2 = x^2+6x+9$

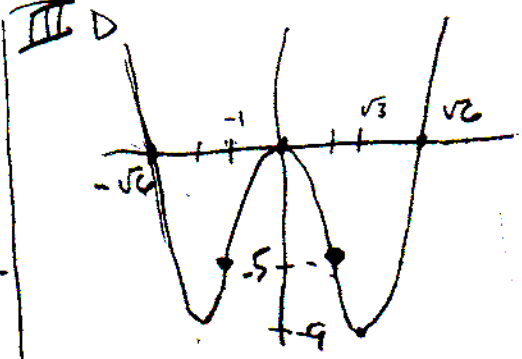
$$\lim_{h\to 0} \frac{(x+h)^2+6(x+h)+9 - x^2-6x-9}{h}$$

$$\lim_{h\to 0} \frac{x^2+2xh+h^2+6x+6h+9-x^2-6x-9}{h}$$

$$\lim_{h\to 0} \frac{2xh+h^2+6h}{h}$$

$$\lim_{h\to 0} 2x+h+6 = 2x+6 = f'(x)$$

III D



III $f(x) = x^4-6x^2$

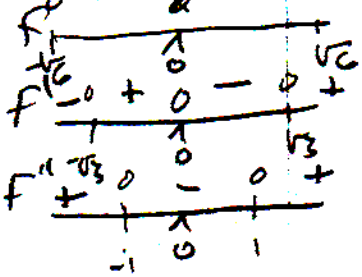
A) $x^2(x^2-6)$

$f'(x) = 4x^3-12x$

$4x(x^2-3)$

$f''(x) = 12x^2-12$

$12(x-1)(x+1)$



B) $x = \{0, \pm\sqrt{3}\}$

max +0- $(0,0)$ $\quad 9-18 = -9$

min -0+ $(\sqrt{3}, -9)$ $(-\sqrt{3}, -9)$ min -0+

C) $f(\pm 1) = 1-6 = -5$

$(1,-5)$ $(-1,-5)$

V

a) $\lim_{x\to 9} \dfrac{6-2\sqrt{x}}{x-9}$

$\lim_{x\to 9} \dfrac{2(3-\sqrt{x})(3+\sqrt{x})}{(x-9)(3+\sqrt{x})}$

$\lim_{x\to 9} \dfrac{2(9-x)}{(x-9)(3+\sqrt{x})}$

$\lim_{x\to 9} \dfrac{-2}{3+\sqrt{x}} = \dfrac{-2}{6} = -\dfrac{1}{3}$

b) $\lim_{x\to 0}\left[\dfrac{3+2x}{x+5x^2} - \dfrac{3}{x}\right]$

$\lim_{x\to 0}\left[\dfrac{3+2x-3(1+5x)}{x(1+5x)}\right]$

$\lim_{x\to 0}\left[\dfrac{3+2x-3-15x}{x(1+5x)}\right]$

$\lim_{x\to 0}\left[\dfrac{-13}{1+5x}\right] = -13$

c) $\lim_{x\to -2} \dfrac{\sqrt{x^2-2}}{x+2}$

$\lim_{x\to -2} \dfrac{-x-2}{x+2}$

$\lim_{x\to -2} \dfrac{-(x+2)}{x+2} = -1$

Extra. $f(x) = x^3 - 2x^2 - 2x$

	1	−2	−2	0
1	1	−1	−3	−3
1	1	0	−3	

pt $(1, -3)$

$m_{\perp}$ $\frac{1}{3}$

$y+3 = \frac{1}{3}(x-1)$

$y = \frac{1}{3}x - \frac{10}{3}$

86

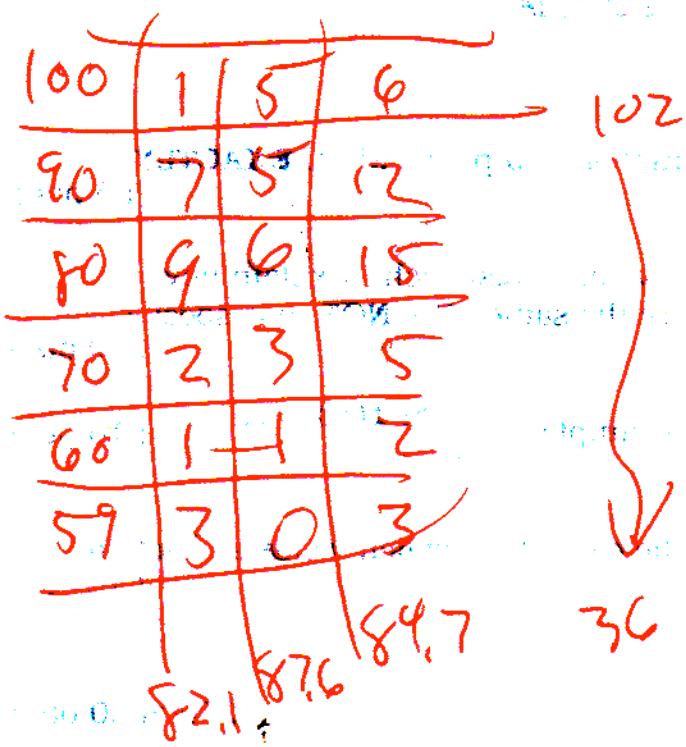
	2	5	T
100	4	2	6
90	4	9	13
80	6	5	11
70	5	2	7
60	1	0	1
54	2	4	6
	84.14	84.81	84.48

105

4

Excellent exam.

Question 13 is a bit weak — probably could use a better question there


100 1 5 6
90 7 5 12
80 9 6 15
70 2 3 5
60 1 1 2
59 3 0 3
102
36
82.1
87.6
84.7

HONORS ANALYSIS
Convince ME that YOU know the concept !
Copy original problem. SHOW ALL WORK !

NAME Analysis Ch 3 1990

PER _____ DATE _____

CHAPTER 3 EXAM

I Explain 'Ums

A) When you have found the derivative of a function at a point, what, **EXACTLY**, have you found? (5 pts)

B) There are two "definitions of the derivative" we use. List each. Explain how they are different AND explain how they are the same. Do **NOT** use specific examples. (10 pts)

C) Explain completely: (Do NOT use specific examples.) $\lim_{x \to a} f(x) = f(a)$ (10 pts)

II Given: $f(x) = (x+3)^2$ Using the "definition of the derivative" which yields a function, find $f'(x)$. (10 pts)

III Given: $f(x) = |x-2|$ (Total 20 pts)

A) Prove f is continuous at x = 2.

B) Prove $f'(2)$ DNE

IV Given: $f(x) = x^4 - 6x^2$ (Total 30 pts)

A) Find f' and f''. Draw appropriate "number line" graphs.

B) Find the COORDINATES of the critical points of f. Identify them as Max, Min or Points of Inflection. **Justify** your responses.

C) Determine the coordinates of any PI which are NOT critical values.

D) Sketch the graph of f.

V Determine the following limits: (5 pts ea)

A) $\lim_{x \to 9} \dfrac{6 - 2\sqrt{x}}{x - 9}$ B) $\lim_{x \to -2} \left[\dfrac{3+2x}{x+5x^2} - \dfrac{3}{x} \right]$ C) $\lim_{x \to -2} \dfrac{\sqrt{x^2} - 2}{x+2}$

EXTRA CREDIT 5 pts

Given : $f(x) = x^3 - 2x^2 - 2x$

Find the equation of the line perpendicular to f (x) at the point with abscissa = 1 .

	2	5	7
100	1	1	2
90	3	6	9
80	5	6	11
70	6	3	9
60	1	1	2
57	4	4	8

102

39

5th

1453/19 = 76.47

1697/21 = 80.81

3150/75 = 78.75

On HW 3,

I took 10 if no attempt at using $\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$ was used.

5 if only 1/2 of def applied (from below increment zero)

Need to emphasize must use def of derivative

① Phrase "using the def..."

or

② when trying to "Prove" a der DNE or exists at a point.

No changes in Exam required.

Honors Analysis Name ANL Ch 3

Copy original problem. Per _____ Date 3/31/94

Convince *me* that you understand the concept!

Chapter 3 Exam

I Given $f(x)$, find $f'(x)$: (5 pts ea)

a) $\left(4x^3 - 5x\right)^2$ b) $x^{\pi} + 3$ c) $4^2 - 1$

II Using the "definition of the derivative" which yields a function, find $f'(x)$: $f(x) = (x-3)^2$ (15 pts)

III Given $f(x) = \begin{cases} x^2 & ; \ x \le 1 \\ 2x - 1 & ; \ x > 1 \end{cases}$ (15 pts ea)

a) ***Prove*** $f(x)$ is or is not continuous at $x = 1$. b) ***Prove*** $f'(1)$ does or does not exist.

IV Given: $f(x) = x^4 - 4x^3$ (30 pts tot)

a) Find $f'(x)$ and $f''(x)$. Draw appropriate "number line" graphs.

b) Find the *coordinates* of the critical points. Identify them as max, min or P.I.

c) Determine the *coordinates* of any PI which are NOT critical values.

d) Specifically *state the interval*(s) where $f(x)$ is concave up.

e) Specifically *state the interval*(s) where $f(x)$ is decreasing.

f) Sketch the graph of $f(x)$.

V Determine the following limits: (5 pts ea)

a) $\lim_{x \to 2} \dfrac{\sqrt{x^2 + 5} - 3}{x - 2}$ b) $\lim_{x \to 0} \sqrt{1 + \dfrac{1}{x}} - \sqrt{\dfrac{1}{x}}$ c) $\lim_{x \to 1} \left[\dfrac{1}{1 - x^3} - \dfrac{1}{3(1 - x)} \right]$

Extra Credit ---------- 5 pts ----------

Given $f(x) = ax^3 + bx^2$.

Determine a and b so the graph of $f(x)$ will have a point of inflection at (1, 2).

	5	6	T
100	0	0	0
90	4	4	8
80	2	1	3
70	4	3	7
60	4	5	9
59	10	8	18

99

25

$\frac{1590}{24}$ 66.3

$\frac{1354}{21}$ 64.5

$\frac{2944}{45}$ 65.4

Excellent exam — Lousy students —

They wait to do problems until they can copy from the boards. and apparently don't learn from seeing the problems done on the board.

I don't know what I'll do to improve the results — I will NOT give back points —

It is time these students found out their true quality level!

make changes as noted on opposite side.

use Part I for final.

Honors Analysis Name AP L ch3

Copy original problem. Per _____ Date 3/29/96

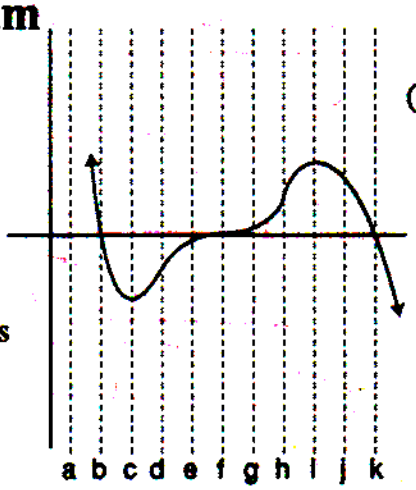
Convince *me* that you understand the concept!

No Calculators.

Chapter 3 Exam

I

Given the graph on the right: (20 pts tot)

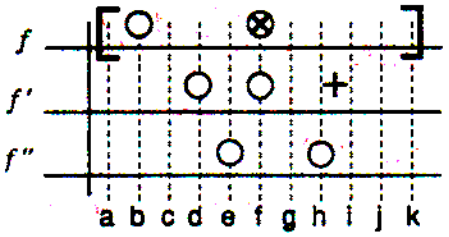


a) Assume the given graph is $f(x)$.
Construct and properly fill in the f f' f'' numberlines.

b) Assume the given graph is $f'(x)$.
Draw an axis. Sketch $f(x)$. Specifically state coordinates of all extrema and PI. Justify, of course.

II

Given:



(30 pts tot)

Note: If you need help filling in the f f' f'' number line on the left, bring your exam paper up and I will fill it in for you. Price: 10 points.

a) Sketch the *graphs* for f f' f'' Three of

b) Specifically state the coordinates of all extrema and points of inflection on f. Justify, of course.

c) Specifically identify extrema on $f'(x)$. Justify.

III

Given $f(x) = \dfrac{x}{x-1}$ (20 pts tot)

a) Using the definition of the derivative which yields a function, find $f'(x)$.

b) Using your answer to part a , find $f'\left(\frac{3}{2}\right)$.

IV

Given $f(x) = \begin{cases} |x+2| & \text{if } -\frac{5}{2} \le x < -1 \\ 2-x^2 & \text{if } -1 \le x < 1 \\ 3-2x & \text{if } 1 \le x \le \frac{5}{2} \end{cases}$ (30 pts tot)

Re letter Thm

a) **Prove** f is continuous at $x = -1$

b) **Prove** f is continuous at $x = 1$

b c) **Prove** $f'(-2)$ does or does not exist.

d e) **Prove** $f'(-1)$ does or does not exist.

c e) **Prove** $f'(1)$ does or does not exist.

just all 5 pts. do not use 2 columns.

Extra Credit ---------- 5 pts ----------

Sketch the graph of the function in section III.

1997

100	2
90	2
80	10
70	5
60	3
59	1

100

55

1878 / 23 81.3

Lowest scores are the usual pretenders —

this was the same exam as last year ... Just in time for parent conferences.

Excellent exam!

make changes shown on opposite side.

Honors Analysis Name AnL Ch3

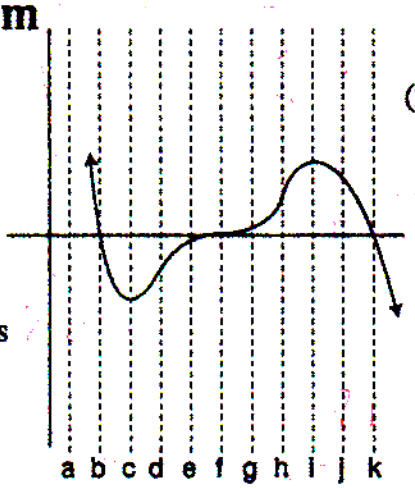
Copy original problem. Per _____ Date 97

Convince *me* that **you** understand the concept!

No Calculators.

Chapter 3 Exam

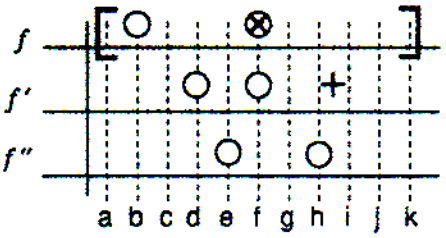
I Given the graph on the right: (20 pts tot)



a) Assume the given graph is $f(x)$.
Construct and properly fill in the f f' f'' numberlines.

b) Assume the given graph is $f'(x)$.
Draw an axis. Sketch $f(x)$. Specifically state coordinates of all extrema and PI. Justify, of course.

II Given:



(30 pts tot)

Note: If you need help filling in the f f' f'' number line on the left, bring your exam paper up and I will fill it in for you. Price: 10 points.

a) Make a single y–axis and three x–axes. Properly label them f, f', and f''. Sketch all three *graphs*

b) Specifically state the coordinates of all extrema and points of inflection on f. Justify, of course.

c) Specifically identify extrema on $f'(x)$. Justify.

III Given $f(x) = \dfrac{x}{x-1}$ (20 pts tot)

a) Using the definition of the derivative which yields a function, find $f'(x)$.

b) Using your answer to part a , find $f'\left(\frac{3}{2}\right)$

IV Given $f(x) = \begin{cases} |x+2| & \text{if } -\frac{5}{2} \le x < -1 \\ 2 - x^2 & \text{if } -1 \le x < 1 \\ 3 - 2x & \text{if } 1 \le x \le \frac{5}{2} \end{cases}$ (30 pts tot)

a) **Prove** f is continuous at $x = -1$

b) **Prove** f is continuous at $x = 1$

c) **Prove** $f'(1)$ does or does not exist.

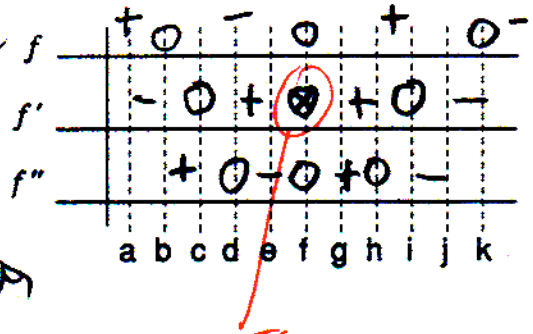
d) **Prove** $f'(-2)$ does or does not exist.

e) **Prove** $f'(-1)$ does or does not exist.

Extra Credit ---------- **5 pts** ----------

Sketch the graph of the function in section III.

I



2 pts

b) 16 pts

max $(b, f(b))$ f' test

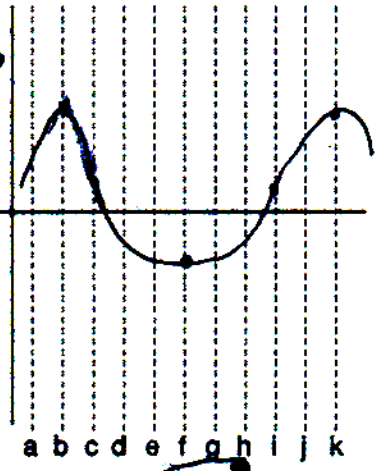
PI $(c, f(c))$ con concave dn to concave up

min $(f, f(f))$ f' test

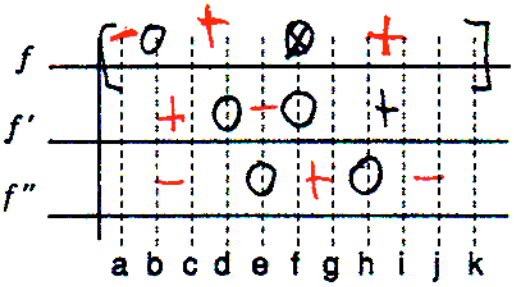
PI $(I, f(I))$ con up to con down

max $(K, f(K))$ f' test

2 ea



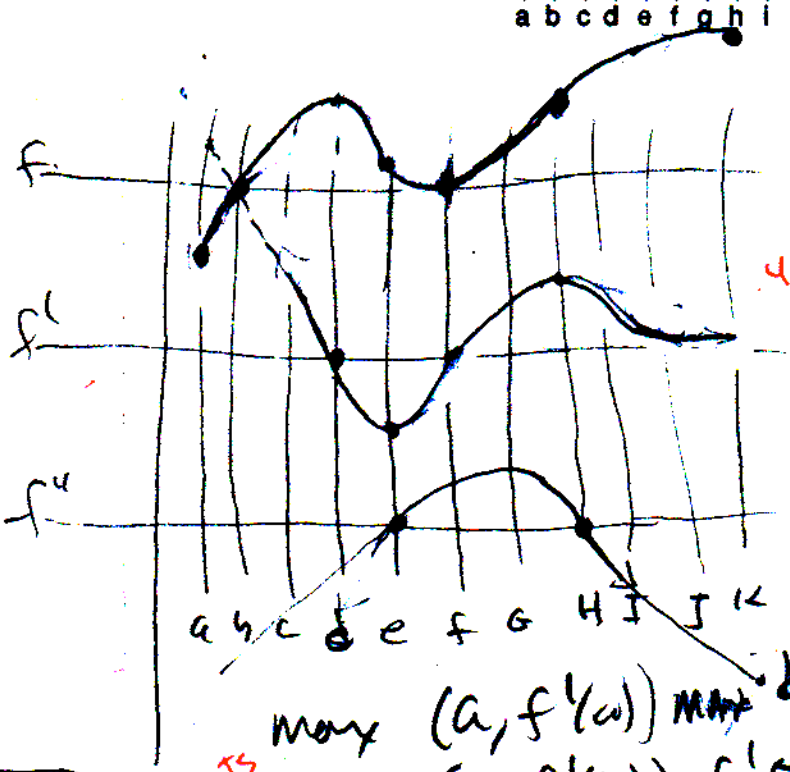
II



Note: max at $(d, f(d))$ and zero at $(b, f(b))$ ∴

① $f < 0$ $x = a$ so max in > 0 and

② $b \Rightarrow$ max so $f''(b) < 0$



4 pts ea

c) 5 pts

max $(a, f'(a))$ max inc fun it

min $(e, f'(e))$ f' test

min K, $f'(K)$ min dec fun it

max $(H, f(H))$ f' test.

b)

$(d, f(d))$ max f' test

$(e, f(e))$ PI down to up.

$(f, f(f))$ min f' test

$(h, f(h))$ PI up to down

$a, f(a)$ min inc fun it

$K, f(K)$ max inc to it

III $f(x) = \frac{x}{x-1}$

a) $\lim_{h \to 0} \frac{1}{h}\left[\frac{x+h}{x+h-1} - \frac{x}{x-1}\right]$

$\lim_{h \to 0} \frac{1}{h}\left[\frac{x^2+hx-x-h-x^2-hx+x}{(x+h-1)(x-1)}\right]$

$\lim_{h \to 0} \frac{1}{h}\left[\frac{-h}{(x+h-1)(x-1)}\right] = \frac{-1}{(x-1)^2}$

$f'\left(\frac{3}{2}\right) = \frac{-1}{\left(\frac{3}{2}-1\right)^2}$

$-\frac{1}{\frac{1}{4}}$

-4

IV

$$f(x) = \begin{cases} |x+2| & \text{if } -\frac{5}{2} \le x < -1 \\ 2 - x^2 & \text{if } -1 \le x < 1 \\ 3 - 2x & \text{if } 1 \le x \le \frac{5}{2} \end{cases}$$

a) $f(-1) = 2 - 1 = 1$ ①

$\lim_{x \to -1^-} x + 2 = 1$ ②

$\lim_{x \to -1^+} 2 - x^2 = 1$ ②

② = ①

cont at $x = -1$

b) $f(1) = 3 - 2 = 1$ ①

$\lim_{x \to 1^-} 2 - 1 = 1$ ②

$\lim_{x \to 1^+} 3 - 2 = 1$ ②

① = ② = ③

f is cont at $x = 1$

c) $\lim_{x \to -2} \frac{|x+2|}{x+2}$

$\lim_{x \to -2^-} \frac{-(x+2)}{x+2} = -1$

$\neq$

$\lim_{x \to -2} \frac{x+2}{x+2} = 1$

$\therefore$ limit DNE ⓛ ≠ ⓡ

$\therefore$ $f'(-2)$ DNE

d) $\lim_{x \to -1^-} \frac{|x+2| - 1}{x+1}$

$\lim_{x \to -1^-} \frac{x+2-1}{x+1}$

$\lim_{x \to -1^-} \frac{x+1}{x+1} = 1$

$\neq$

$\lim_{x \to -1^+} \frac{2 - x^2 - 1}{x+1}$

$\lim_{x \to -1} \frac{1 - x^2}{x+1}$

$\lim_{x \to -1} 1 - x = 2$

$\therefore$ limit DNE ⓛ ≠ ⓡ

$\therefore$ $f'(-1)$ DNE.

e) $\lim_{x \to 1^-} \frac{2 - x^2 - 1}{x - 1}$

$\lim_{x \to 1^-} \frac{1 - x^2}{x - 1}$

$\lim_{x \to 1^-} \frac{-(1+x)}{x-1}$

-2

$\lim_{x \to 1^+} \frac{3 - 2x - 1}{x - 1}$

$\lim_{x \to 1^+} \frac{2 - 2x}{(x-1)}$

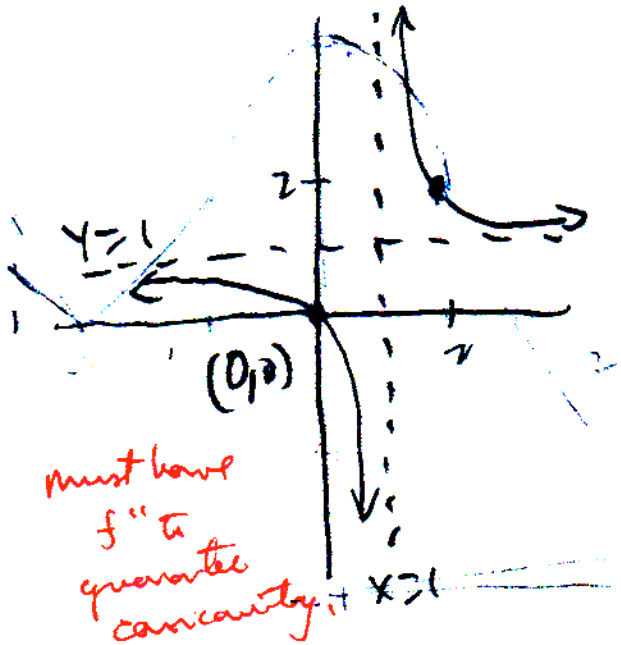
$\lim_{x \to 1^+} \frac{-2(x-1)}{(x-1)}$

-2

$=$

$f'(1)$ exists and is equal to -2.

extra.



must have f'' to guarantee concavity

Great exam

2 Typos in ~~[illegible]~~

3/27/98

104

53

100	3
90	7
80	3
70	6
60	3
54	1

$\frac{1843}{22} = 83.8$

Steven Sogge 95

Garth Gross 66

Stephanie Angelil 60

Eric Galentine 49

I $f(x) = \begin{cases} kx^2 & \text{if } x \le 2 \\ 2x+k & \text{if } x > 2 \end{cases}$

a) $f(2) = 4k$ ① $= \frac{16}{3}$

$\lim_{x\to 2^-} f(x) = 4k$

$\lim_{x\to 2^+} f(x) = 4+k$ ③

$\frac{16}{3}$

$4k \overset{\text{set}}{=} 4+k$

$3k = 4$

$k = \frac{4}{3}$

So $\lim_{x\to 2^-} f(x) = \frac{16}{3}$

$\lim_{x\to 2^+} f(x) = \frac{16}{3}$ ② $\frac{16}{3}$

b) $f(x) = \begin{cases} \frac{4}{3}x^2 & \text{if } x \le 2 \\ 2x + \frac{4}{3} & \text{if } x > 2 \end{cases}$

c) $\lim_{x\to 2^-} \frac{\frac{4}{3}x^2 - \frac{16}{3}}{x-2}$

$\lim_{x\to 2^-} \frac{\frac{4}{3}(x-2)(x+2)}{(x-2)}$

$\frac{4}{3}\cdot 4 = \frac{16}{3}$

$\lim_{x\to 2^+} \frac{2x + \frac{4}{3} - \frac{16}{3}}{x-2}$

$\lim_{x\to 2^+} \frac{2x-4}{x-2}$

$\lim_{x\to 2^+} \frac{2(x-2)}{x-2} = 2$

$\frac{16}{3} \neq 2 \therefore$ limit DNE

⊥ f(2)

Since limit is def ab Dn, Dn DNE.

II $f(x) = \frac{x^2-1}{x} = x - \frac{1}{x}$

$\lim_{h\to 0} \frac{1}{h}\left(x+h - \frac{1}{x+h} - x + \frac{1}{x}\right)$

$\lim_{h\to 0} \frac{1}{h}\left(\frac{1}{x} - \frac{1}{x+h} + h\right)$

$\lim_{h\to 0} \frac{1}{h}\left(\frac{x+h-x+xh(x+h)}{x(x+h)}\right)$

$\lim_{h\to 0} \frac{1}{h}\left(\frac{h+x^2h+xh^2}{x(x+h)}\right)$

$\lim_{h\to 0} \frac{1+x^2+xh}{x(x+h)}$

$f'(x) = \frac{1+x^2}{x^2}$

Extra

Jake 1 Lap = 40 sec

$\frac{1}{40}$ Lap = 1 sec.

they meet every 15 secs.

$\frac{15}{40}$ Lap = 15 sec = Jake

$\frac{25}{40}$ Lap = Jed = $\frac{5}{8}$

So Jed $\frac{5}{8}$ Lap = 15 sec.

1 Lap = $\frac{3}{5} \cdot 40$

= 24 sec.

III $f(x) = x^3 - 3x + 2$

a)
	1	0	−3	+2
1	1	1	−2	0

$x^2 + x - 2$

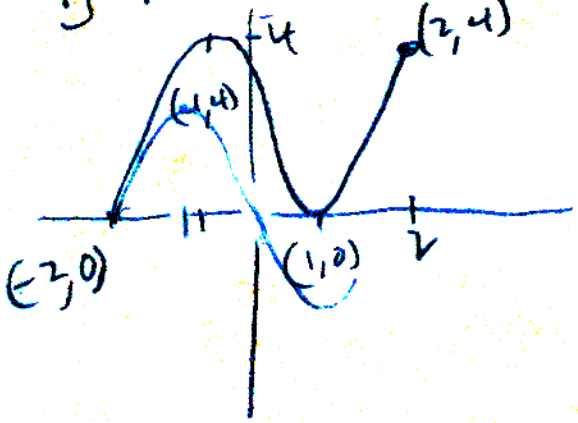
$(x+2)(x-1)$

$(x-1)^2(x+2)$

x intercepts 1, −2

y intercept 2

	1	0	−3	2
−2	1	−2	1	0
−1	1	−1	−2	4
2	1	2	1	4
1	1	0	−3	0



b) $f'(x) = 3x^2 - 3$

$3(x-1)(x+1)$

f' : + 0 − 0 +

−1 1

f increasing $-2 \le x \le -1$ or $1 \le x \le 2$

c) (−2, 0) ENDPT MIN INC from it

(−1, 4) max f' changes

(1, 0) min f' changes

(2, 4) ENDT MAX INC to it.

d) $f'' = 6x$: − 0 + (0)

PI (0, 2) f changes from down to up.

IV a) $\lim_{x \to 4} \left(\frac{1}{x} - \frac{1}{4}\right)$

$\frac{1}{4} - \frac{1}{4} = 0$

b) $\lim_{x \to 4} \left[\left(\frac{1}{x} - \frac{1}{4}\right)\left(\frac{1}{x-4}\right)\right]$

$\lim_{x \to 4} \left[\left(\frac{4-x}{4x}\right)\left(\frac{1}{x-4}\right)\right] = \frac{-1}{16}$

c) $\lim_{x \to 4} \left[\left(\frac{1}{x} - \frac{1}{4}\right)(x-2)\right]$

$0 \cdot (2)$

0

d) $\lim_{x \to 4} \left[\left(\frac{1}{x} - \frac{1}{4}\right)\left(\frac{1}{x-4}\right)^2\right]$

$\lim_{x \to 4} \left[\frac{4-x}{4x} \cdot \frac{1}{x-4} \cdot \frac{1}{x-4}\right]$

$\lim_{x \to 4^-} \left[\frac{-1}{4x(x-4)}\right] \to +\infty$ ∴ No limit.

Honors Analysis Name AnL Ch 3

Copy original problem. Per _____ Date 3/27/98

Convince *me* that **you** understand the concept!

Chapter 3 Exam

Became Ch3 make up Exam

I Given $f(x) = \begin{cases} kx^2 & \text{if } x \le 2 \\ 2x + k & \text{if } x > 2 \end{cases}$ (30 pts tot)

a) Determine k so that $f(x)$ continuous. (Hint: Use the definition of continuity.)

b) Rewrite $f(x)$ using the value of k found in the previous step.

c) Prove $f'(2)$ does not exist.

II *Using the definition of the derivative* which yields a function,

find $f'(x)$ given $f(x) = \dfrac{x^2 - 1}{x}$. (15 pts)

III Given $f(x) = x^3 - 3x + 2 \;;\; x \in [-2, 2]$ (40 pts tot)

a) State the coordinates of all *axis* intercepts of $f(x)$.

b) State the intervals (using proper notation) where $f(x)$ is increasing; decreasing.

c) Identify and state the coordinates of all extrema. **Justify.**

d) State the coordinates of any Points of Inflection. **Justify.**

e) Draw an accurate graph of $f(x)$. Specifically show all coordinates found above.

IV Find the following limits. (If they do not exist, explain fully how you know that.) (15 pts tot)

a) $\lim\limits_{x \to 4} \left(\frac{1}{x} - \frac{1}{4}\right)$ b) $\lim\limits_{x \to 4} \left[\left(\frac{1}{x} - \frac{1}{4}\right)\left(\frac{1}{x - 4}\right)\right]$

c) $\lim\limits_{x \to 4} \left[\left(\frac{1}{x} - \frac{1}{4}\right)(x - 2)\right]$ d) $\lim\limits_{x \to 4} \left[\left(\frac{1}{x} - \frac{1}{4}\right)\left(\frac{1}{x - 4}\right)^2\right]$

Extra Credit ---------- **5 pts** ----------

Jake can run around a circular track in forty secons. Jed, running in the opposite direction, meets Jake every fifteen seconds. What is Jed's time to run aroujnd the track, expressed in seconds?

$f(x) = \frac{1}{x+1}$

I

A) $\lim_{h\to 0} \frac{\frac{1}{x+1+h} - \frac{1}{x+1}}{h}$

$\lim_{h\to 0} \frac{1}{h}\left[\frac{x+1-(x+1+h)}{(x+1)(x+1+h)}\right]$

$\lim_{h\to 0} \frac{1}{h}\left[\frac{-h}{(x+1)(x+1+h)}\right]$

No Squared term -1

$\frac{-1}{(x+1)^2}$

B) $f'(2)$ $f(x) = \frac{1}{x+1}$

$\lim_{x\to 2} \frac{\frac{1}{x+1} - \frac{1}{3}}{x-2}$

$\lim_{x\to 2} \frac{3-(x+1)}{3(x+1)(x-2)}$

$\lim_{x\to 2} \frac{3-x-1}{3(x+1)(x-2)}$

$\lim_{x\to 2} \frac{-(x-2)}{3(x+1)(x-2)}$

$\frac{-1}{3\cdot 3}$

$-\frac{1}{9}$

II

$f(x) = 3(x-1)^2$

$f = 3x^2 - 6x + 3$

	3	−6	3
2	3	0	3
2	3	6	

$(2,3)$ $m = 6$

$y - 3 = 6(x-2)$

$y = 6x - 9$

No line -5

III

A) $\lim_{x\to 3} \frac{x^2-3x}{x^2-4x+3}$

$\lim_{x\to 3} \frac{x(x-3)}{(x-3)(x-1)}$

$\frac{3}{2}$

B) $\lim_{x\to 2} \frac{x^3-8}{x-2}$

$\lim_{x\to 2} \frac{(x-2)(x^2+2x+4)}{x-2}$

$4+4+4$

12

No diff of cubes -4

IV

$$f(x) = \begin{cases} x^2 & 0 \le x \le 1 \\ ax+3 & 1 < x < 2 \\ x^2-6 & 2 \le x \le 3 \end{cases}$$

doesn't have lim -15

has lim but little else -10

$\lim_{x\to 1^-} x^2 = 1$

$\lim_{x\to 1^+} ax+3 = a+3 \overset{\text{set}}{=} 1$

$a = -2$

$\lim_{x\to 2^-} 2a+3 \Big|_{a=-2} = -1$

$\lim_{x\to 2^+} x^2-b = 4-b \overset{\text{set}}{=} -1$

$b = 5$

4 $f(x) = x(x-3)^2$

$f(x) = x^3 - 6x^2 + 9x$

$f'(x) = 3x^2 - 12x + 9$

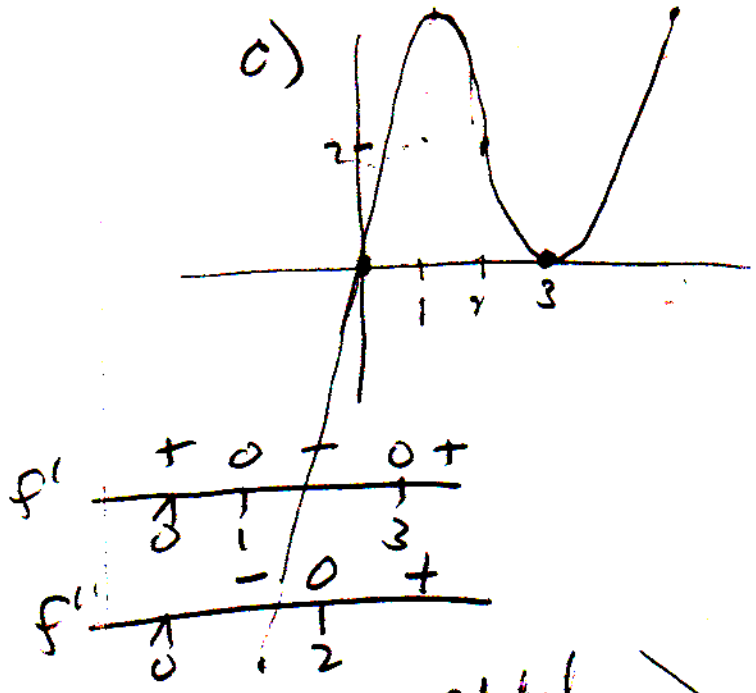
$3(x^2 - 4x + 3)$

$3(x-3)(x-1)$

$f''(x) = 6x - 12$

$6(x-2)$

c)



$f(1) = 1(1-3)^2$ $(1, 4)$ max f' test Global

min f' test local

$f(3) = 0$ $(3, 0)$ a

$f(2) = 2(2-3)^2 \Rightarrow (2, 2)$ PI

b) $f(2) = 2(2-3)^2 \Rightarrow (2, 2)$ PI

$f(-1) = -1(-4)^2$ $(-1, -16)$ min INC to it global

$f(4) = 4(4-3)^2 = (4, 4)$ max INC to it global

ENDPTS -4

EXTRA

JACK 1 LAP = 40 sec

$\frac{1}{40}$ LAP = 1 sec.

they meet every 15 sec.

JACK $\frac{15}{40}$ = 15 sec

$\frac{3}{8}$ of a LAP ∴ Jill $\frac{5}{8}$ LAP = 15 sec

1 LAP = $\frac{8}{5} \cdot 15$

= 24 sec.

92

100	0	1	1
90	2	5	7
80	5	2	7
70	3	4	7
60	1	5	6
59	1	5	6
	81,5	71,9	77,2
		1573	
	897		2470

103

24

3/26/98

100	4	6	105
90	4	3	
80	3	4	
70	2	3	
60	3	3	
59	3	0	

1535 / 19 = 80,8

1755 / 20 = 87,8

38

3290 / 39 = 84,4

Excellent exam. tests items taught! the pretenders are being exposed.

change spelling as indicated.

Some comments

Last years exam had some extra credit so scores are up maybe 3 pts overall but still the students did pretty well.

Honors Analysis
Copy original problem.
Convince *me* that **you** understand the concept!

Name ANALYSIS Ch 3

Per _____ Date 4/6/92

NO calculators

3/26/99

Chapter 3 Exam

I Definition of the derivative: (10 pts ea)

a) Using the *definition of the derivative* which yields a **function**, find $f'(x)$ given $f(x)=\frac{1}{x+1}$.

b) Using the *definition of the derivative* which yields a **number**, find $f'(2)$ given $f(x)=\frac{1}{x+1}$.

II Given $f(x)=3(x-1)^2$. Find the equation of the tangent line to $f(x)$ at $x=2$. (10 pts)

III Find the following limits: (5 pts ea)

a) $\lim_{x\to 3}\frac{x^2-3x}{x^2-4x+3}$

b) $\lim_{x\to 2}\frac{x^3-8}{x-2}$

IV Determine a and b so $f(x)$ is continous in the interval [0, 3]. (tot 25 pts)

given: $f(x)=\begin{cases} x^2 & \text{if } 0\le x\le 1 \\ ax+3 & \text{if } 1<x<2 \\ x^2-b & \text{if } 2\le x\le 3 \end{cases}$

V Given $f(x)=x(x-3)^2$ D_f $x\in[-1, 4]$. (tot 35 pts)

a) Determine the coordinates of all extrema. Identify as global or local. Justify!

b) Determine coordinates of all points of inflection.

c) Sketch $f(x)$.

Extra Credit 5 pts

Jack can run around a circular track in forty seconds. Jill, running in the opposite direction, meets Jack every fifteen seconds. What is Jill's time to run around the track, expressed in seconds?

f f' f''

Mr. Mumaugh

I

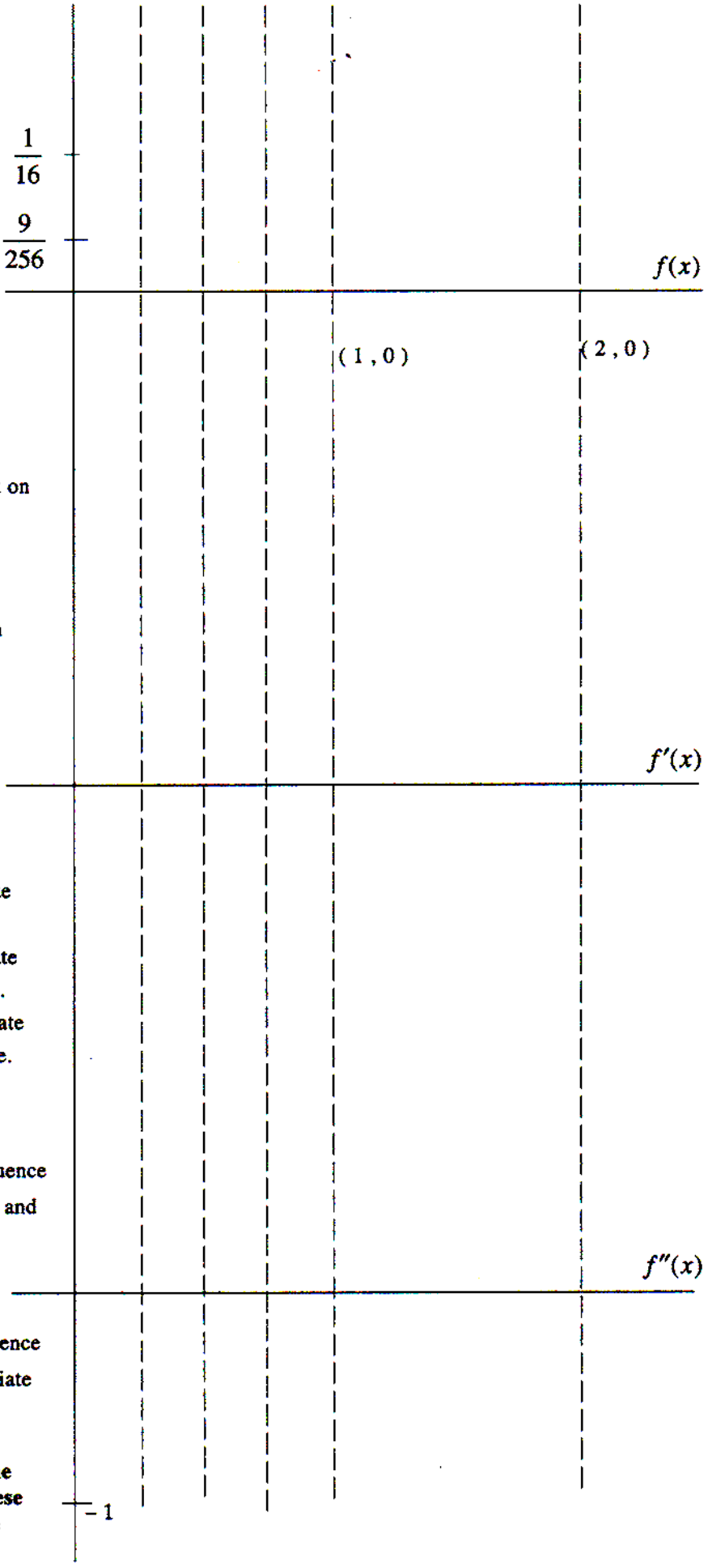
Given: $f(x) = x^4 - 2x^3 + x^2$

$f''(x) = 0 \rightarrow x \in \{\frac{1}{4}, \frac{3}{4}\}$

$f'(\frac{1}{4}) \approx \frac{1}{2}$; $f'(\frac{3}{4}) \approx -\frac{1}{2}$

$f''(x)$ has minimum at $(\frac{1}{2}, -1)$

1. Express $f(x)$ in factored form; determine ***all*** zeros of $f(x)$. Plot those points on the top axis.
2. Find $f'(x)$ and write in factored form.
3. Find all critical values of x. $f'(x) \overset{\text{set}}{=} 0$ and solve. Plot the zeros on the middle axis.
4. Write the **coordinates** of all critical values. Plot on the top axis.
5. Find $f''(x)$. Normally you would find these zeros now. However, the zeros were given above. We shall use them. Plot zeros on bottom axis.
6. Put values found in #5 into $f(x)$. Write coordinates. Plot on top axis.
7. Put values found in #5 into $f'(x)$. Write coordinates. Plot on middle axis.
8. On a number line, locate zeros of $f(x)$. Indicate where $f(x)$ is positive and where it is negative. On a number line, locate zeros of $f'(x)$. Indicate where $f'(x)$ is positive and where it is negative. On a number line, locate zeros of $f''(x)$. Indicate where $f''(x)$ is positive and where it is negative.
9. Sketch all three functions on their axes.
10. Explain how the values (+ or –) of $f''(x)$ influence $f(x)$ and $f'(x)$. i.e. What behavior in $f(x)$, and $f'(x)$ do you associate with $f''(x) > 0$? ; $f''(x) > 0$?
11. Explain how the values (+ or –) of $f'(x)$ influence $f(x)$ i.e. What behavior in $f(x)$do you associate with $f'(x) > 0$? ; $f'(x) > 0$?
12. **The results of the above 11 problems are of the utmost importance !! Write a summary of these results AND your conclusions. Use complete sentences and be VERY complete.**



II

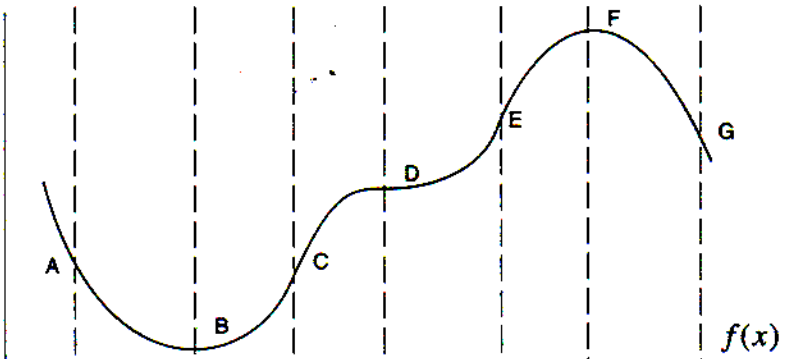
Carefully draw F ' and F " on the graphs.

Show the corresponding points of A, B. C, D, etc

Justify the locations of A', B' C' etc using complete sentences.

Determine the DEGREE of F(x).

(hint : $f'(x)$ and $f''(x)$ are continuous.)



III

Sketch a smooth curve of F (x) illustrating the following :

(Each problem uses a unique axis system and is independent of any other problem.)

a) $f(1)=3,\ f'<0\ \forall\ x<1;\ f'>0\ \forall\ x>1$

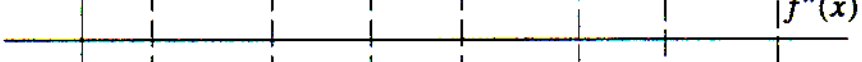
b) $f(1)=3,\ f''<0\ \forall\ x<1;\ f''>0\ \forall\ x>1$

c) $f'>0\ \forall\ x<2;\ f'<0\ \forall\ x>2;\ f(2)=3$

d) $x>0\quad f(1)\stackrel{\epsilon}{=}0;\ f'(x)=\frac{1}{x}$

(hint : What is $f''(x)$ and what does it tell you?)

Use the def of the derivative to find out . . .



IV

Given: $f(x)=\frac{x^2+1}{x}$

$D_f\quad -2\le x\le 2;\ x\neq 0$

A) Find $f'(x)$, using the second definition of the derivative.

B) Find $f''(x)$, using the second definition of the derivative.

C) Make a number line for $f(x)$.

D) Make a number line for $f'(x)$.

E) Make a number line for $f''(x)$.

F) What are the coordinates (of $f(x)$) of the critical values.

G) What are the coordinates of (of $f(x)$) $\big(-2,\ f(-2)\big)$ and $\big(2,\ f(2)\big)$?

H) List the coordinates and specifically identify all extrema.

I) List intervals where $f(x)$ is increasing.

J) List intervals where $f(x)$ is concave up.

K) Sketch $f(x)$

El Quizzo

Name _____

No Calculators

Per _____ Date _____

I Given $f(x)=3x^4-8x^3+6x^2-1$ for $-1\le x\le 2$.

Determine and identify all significant points (ie. extrema, PI, intercepts, etc)

II Find the limit:

a) $\lim_{x\to 0}\left(\frac{3+2x}{x+5x^2}-\frac{3}{x}\right)$

b) $\lim_{x\to 0}\frac{\sqrt[3]{x+1}-1}{x}$

Turn over for part III

III State and identify both "definitions of the derivative."

Explain how they are different.
Explain how they are the same.
Explain when each is used. Be complete.
Demonstrate each definiton of the derivative

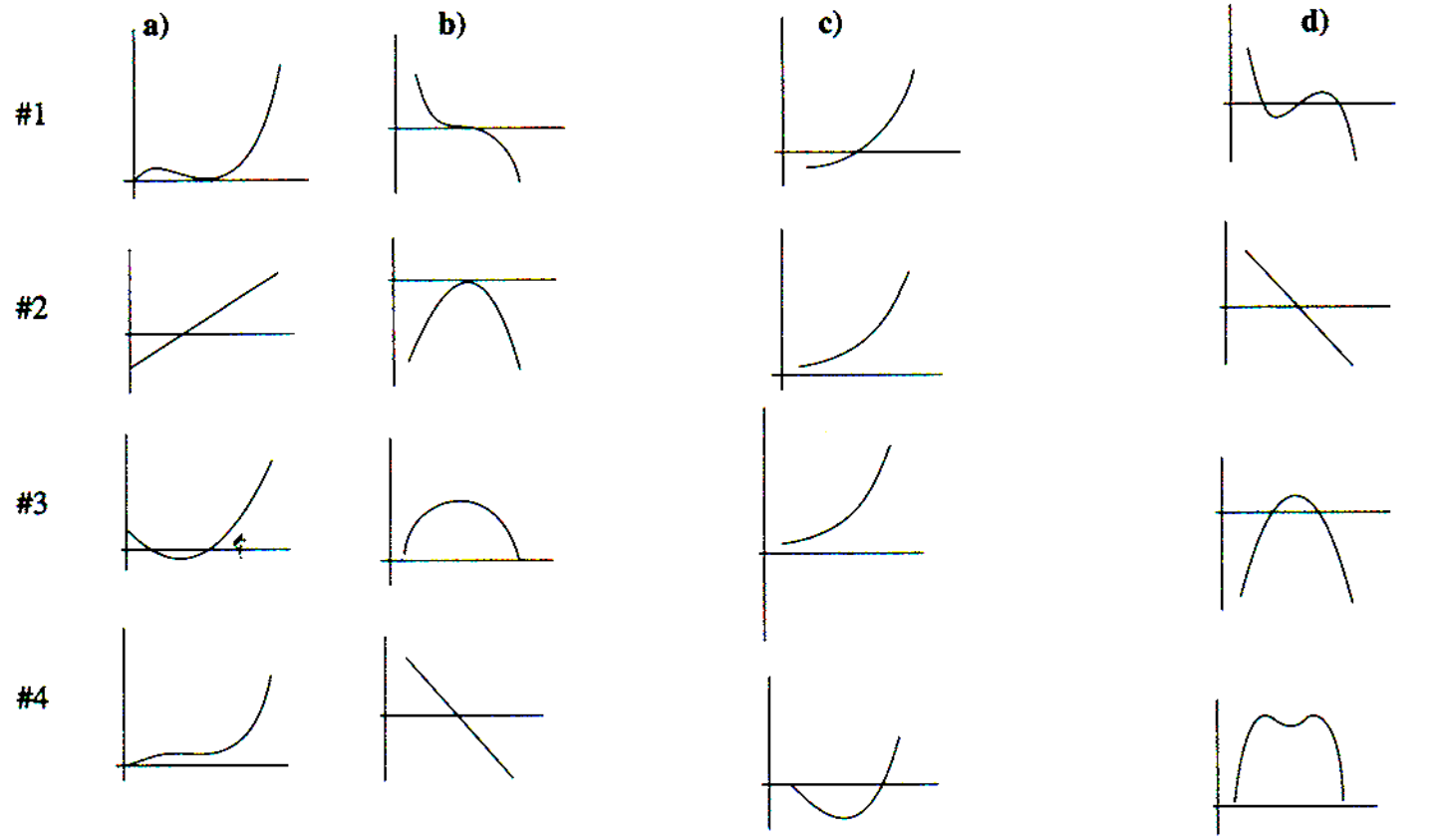
using the function $f(x)=\frac{x^2+1}{x}$ and the point $\left(2, \frac{5}{2}\right)$

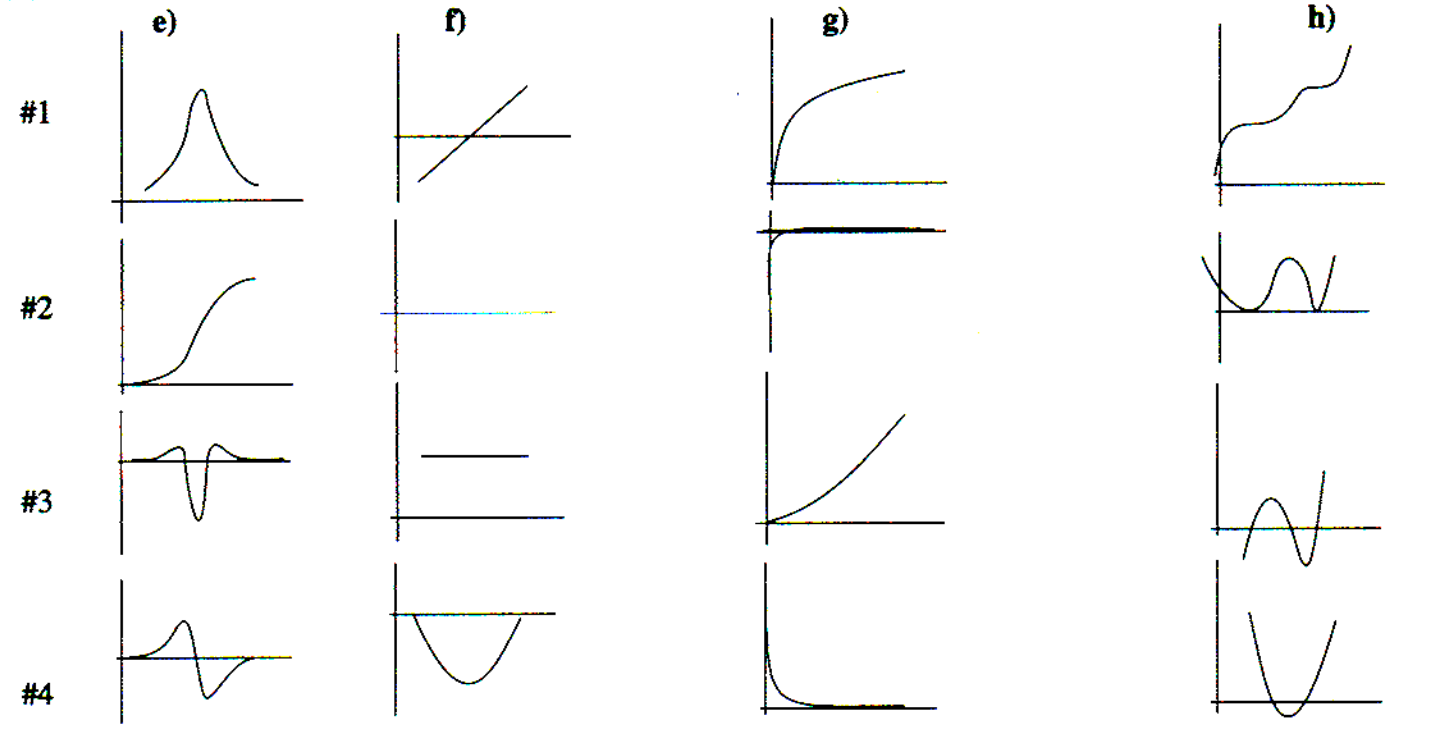
Analysis

J Mumaugh

We want *contenders!* Can you find the *pretender*?

Each of the following columns is headed by the graph of a function $f(x)$. Underneath this graph are three others that are, in random order, the graphs of $f'(x)$, $f''(x)$, and another function. Your job is to determine the pretender and to label the others properly.

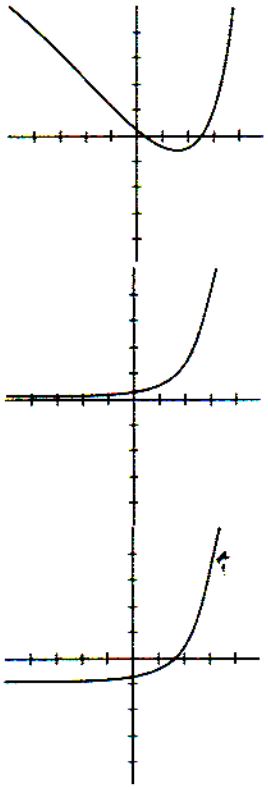




Sherlock Heston Knows his f from his f' and his f''. Do you?

Under each number are 3 axes. The curves, f , f', and f'' are in random order. Your job is to decide which curve is which. Consider problem #1. The first graph is f and it is concave up everywhere so the 2nd curve would be f'' since it is everywhere positive and the third curve changes from negative to positive when f changes from decreasing to increasing so it would be f'.

1



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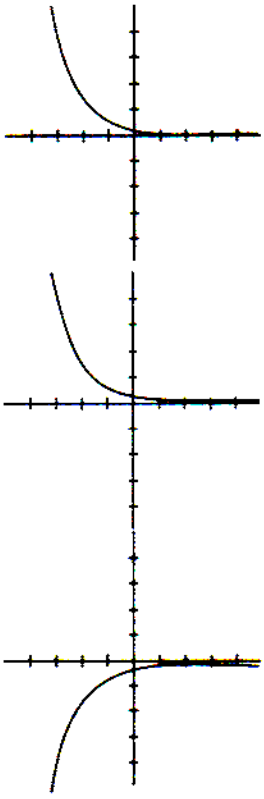

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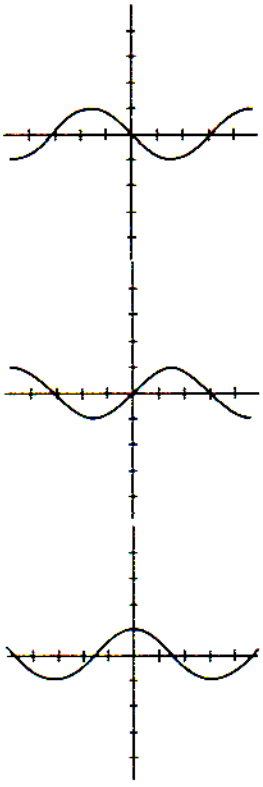

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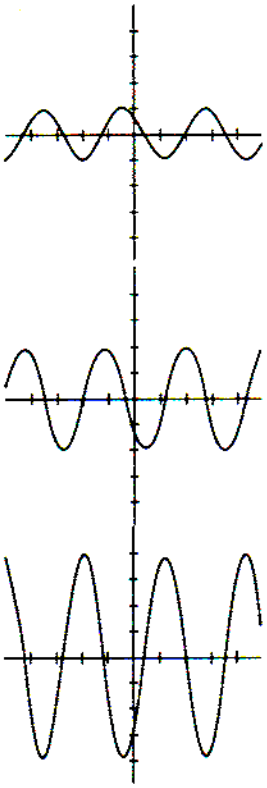

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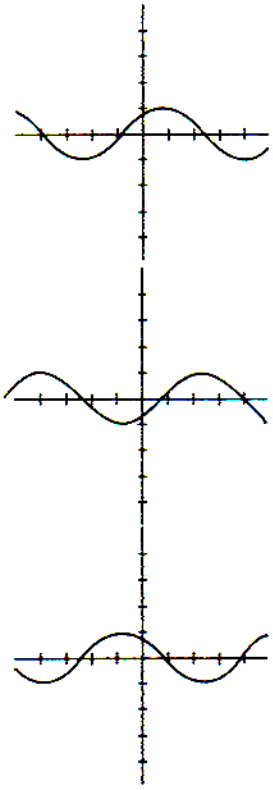

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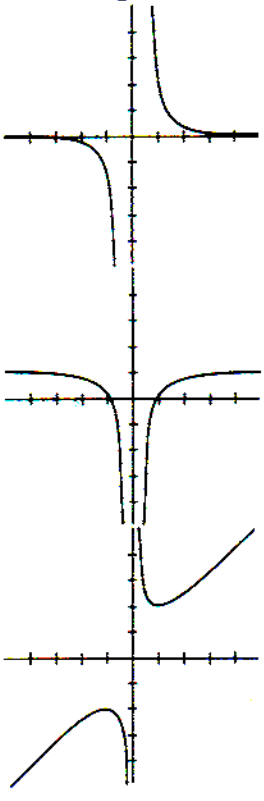

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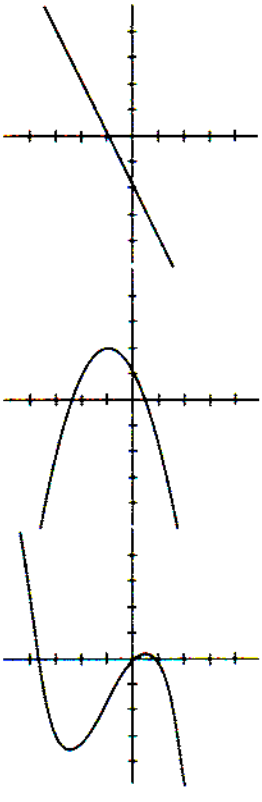

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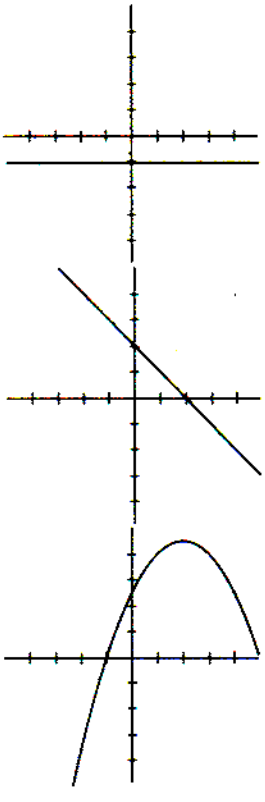

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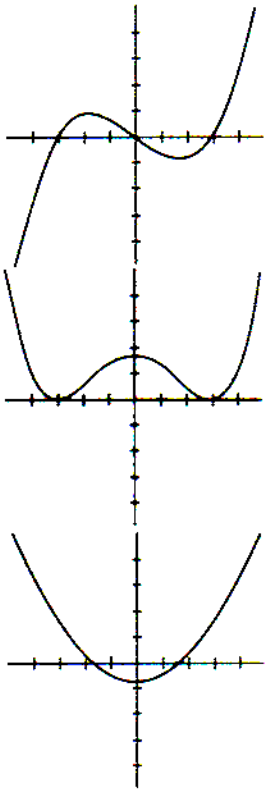

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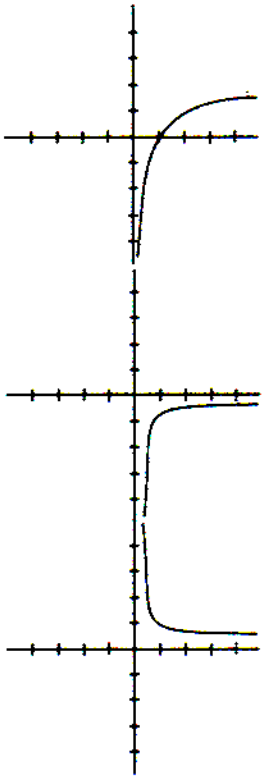


Honors Analysis Chapter 3 Heavy Duty Quiz Name _____

Open Notes NOT open friends **25 minutes** Period _____ Date _____

Do all work and put answers on this paper.

I	**II**	**III**	**IV**

I What is the effect on:

a) $f(x)$ and $f'(x)$ when $f''(x) > 0$

b) $f(x)$ when $f'(x) = 0$

c) $f(x)$ when $f'(x) < 0$

d) $f(x)$ when $f'(x) > 0$

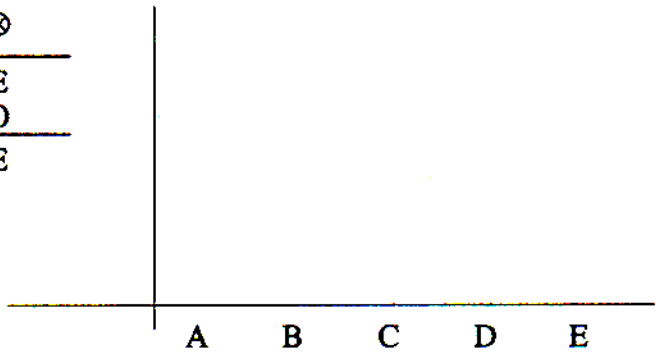
e) $f(x)$ and $f'(x)$ when $f''(x) < 0$

f) $f(x)$ and $f'(x)$ when $f''(x) = 0$

II Given $f'(x)$ and $f''(x)$ number lines. Sketch the graph of a function which has the properties described:

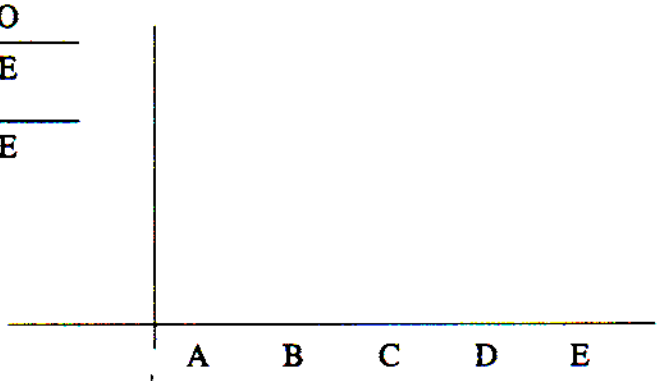
a)

	A	B	C	D	E
$f'(x)$	O		O	+	⊗
$f''(x)$	–	O		O	O
	A	B	C	D	E



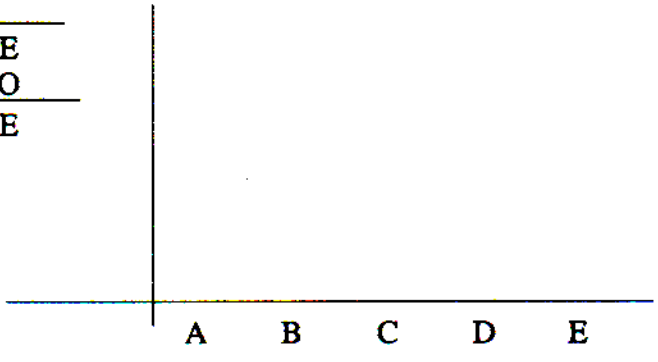
b)

	A	B	C	D	E
$f'(x)$	– O		⊗		O
$f''(x)$	+	O	O	O	
	A	B	C	D	E



c)

	A	B	C	D	E
$f'(x)$			+		
$f''(x)$	O	O	O	–	O
	A	B	C	D	E



Sections III and IV on back.

III Find the limit: $\lim_{x \to 1} \left(\frac{1}{1-x^3} - \frac{1}{3(1-x)} \right)$

IV Given: $f(x) = 3x^4 - 8x^3 + 6x^2 + 1 \quad \forall \quad x \ \varepsilon \ [-1, \ 2]$.
Determine (identify as local or global) the *coordinates* of all extrema and PI. Justify, of course.

Determine if the following statements are true or false. Explain reasoning. A counter-example is acceptable (you still need to explain your method.)

1) $f(x)=\begin{cases}\dfrac{x^2+2x-3}{x+3} & \text{if } x\neq -3\\ -4 & \text{if } x=-3\end{cases}$ $f(x)$ is continuous at $x=-3$.

2) Let $f(x)$ be a function, D_f $x \in R$.

a) If $f(x)$ is differentiable (has a derivative) at $x=2$, then $f(x)$ must be continuous at $x=2$.

b) If $f(x)$ is continuous at $x=2$, then $f(x)$ must be differentiable at $x=2$.

c) If $f(x)$ is continuous at $x=2$, then $\lim\limits_{x\to 2}\big(f(x)-f(2)\big)=0$.

d) If $f(x)$ is continuous, then $|f(x)|$ is continuous. (T)

e) If $|f(x)|$ is continuous, then $f(x)$ is continuous. (F)

f) If $f(x)$ is continuous, then $f(|x|)$ is continuous. (T)

3) If $f'(x)=g'(x)$, then $f(x)=g(x)$.

4) Let $\lim\limits_{x\to a} f(x)=f\left(\lim\limits_{x\to a} x\right)$.

a) $f(x)$ is continuous.

b) $f(x)$ is differentiable.

Provide a detailed reason for each problem:

5) Define continuity. Give the "Dick & Jane" version and the mathematical version. Prove $f(x)=x^3-8$ is continuous at $x=2$.

6) Write the two versions of the definition of the derivative.

a) Explain how they are different.

b) Explain how they are the same.

c) Show a properly labeled graphical representation for each (as was done in lecture).

d) Tell which version is best suited for which purposes.

e) Using each of the versions, find $f'(2)$ when $f(x)=x^3-8$.

7) Given $f(x)=|x-3|$.

a) Prove $f(x)$ is continuous at $x=3$.

b) Prove $f'(3)$ DNE.

8) Draw the graph of a function which illustrates the situation concerning the definition of continuity:

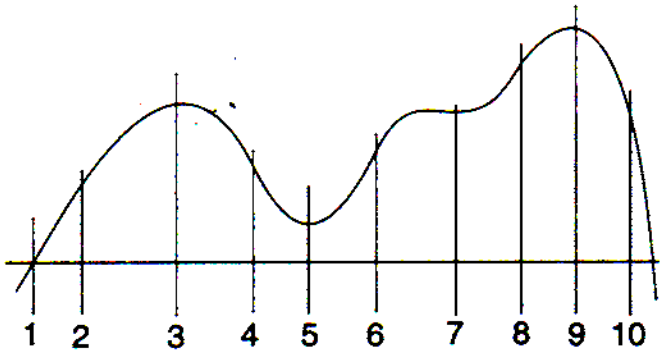
a) Step 1 met. Step 2 met. Step 3 not met.

b) Step 1 not met. Step 2 met.

c) Step 1 not met. Step 2 not met.

d) Step 1 met. Step 2 not met.

9. Use the figure to the right to determine "True" or "False".



a) $f(x)$ changes sign at $x = 1$.

b) $f''(2) = 0$

c) $f'(3) = 0$

d) $f''(4) = 0$

e) $f''(5) = 0$

f) $f''(5) < f''(3)$

g) $f'(7) < 0$

h) $f''(8) = 0$ i) $f(2) < f(7)$ j) $f'(9) = 0$ k) $f'(10) = 10$ l) $f''(10) < 0$

m) $f'(2) < f'(4)$ n) $f'(7) < f''(7)$ o) $f''(2) > f''(4)$

10. Given: $f(x) = \begin{cases} 2x+1 & \text{if } x \in [-2,0) \\ -x+1 & \text{if } x \in (0,2) \\ -1 & \text{if } x = 2 \\ 2x-5 & \text{if } x \in (2,3] \end{cases}$

Note: $x \in [-2, 0)$
means the same thing as
$-2 \le x < 0$

The bracket means "including.."
and the parenthesis means "not including"

a) Prove f is or is not continuous at $x = 0$.

b) Prove f is or is not continuous at $x = 2$.

c) Prove $f'(0)$ does or does not exist.

d) Prove $f'(2)$ does or does not exist.

e) Draw the graph of f.

11. Given: $f(x) = 3x^4 - 4x^3$; $D_f x \in [-1,2]$

a) Determine all zeros of f.

b) Find $f'(x)$ and $f''(x)$

c) Draw number lines for $f'(x)$ and $f''(x)$.

d) Compute the coordinates of all significant points. (Justify, of course)

e) Graph f.

12. Given: $f(x) = \begin{cases} \dfrac{x^3-1}{x-1} & \text{if } x \neq 1 \\ k & \text{if } x = 1 \end{cases}$

a) Determine the value of k so f is continuous.

b) Find $f'(1)$

13. Given: $f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ k & \text{if } x = 1 \\ 2x & \text{if } x > 1 \end{cases}$

a) Determine the value of k so f is continuous.

b) Find $f'(1)$

Mr. Summary

Mr. Mumaugh

Use a calculator where *necessary* (as contrasted by use due to *laziness*).

I The limit concept

A) Explain, compare and contrast the following: $\lim_{x\to a^+} f(x)$, $\lim_{x\to a^-} f(x)$, $\lim_{x\to a} f(x)$, $\lim_{x\uparrow a} f(x)$, $\lim_{x\downarrow a} f(x)$

B) Explain what "If there is no reason not to, we just stuff it in." means and why and where do we use it.

C) Explain how to "simplify" limit problems. Discuss various methods *but do not use* any specific examples.

II Explain the effect on ....

A) $f(x)$ when $f'(x)>0$

B) $f(x)$ when $f'(x)<0$

C) $f(x)$ when $f'(x)=0$

D) $f(x)$ when $f''(x)=0$

E) $f(x)$ when $f''(x)>0$

F) $f(x)$ when $f''(x)<0$

G) $f'(x)$ when $f''(x)<0$

H) $f'(x)$ when $f''(x)>0$

I) $f'(x)$ when $f''(x)=0$

III Given: $f(x)=\dfrac{x^2+16}{x-3}$ and $f'(x)=\dfrac{x^2-6x-16}{x^2-6x+9}$; also $g(x)=x+3$; $h(x)=f(x)-g(x)$

A) Using long division divide the denominator of $f(x)$ into the numerator of $f(x)$.

B) Explain why there is a vertical asymptote for $f(x)$ at $x=3$. (Describe the situation as $x\to 3^-$ for example.) Be complete! Remember, this *is* an honors class so do honors level work!

C) Compute (maybe you could make your *programmable* calculator do this for you?)

1) Find $f(8)$, $f(80)$, $f(800)$, $f(8000)$ and $f(-2)$, $f(-20)$, $f(-200)$, $f(-2000)$.

2) Find $h(8)$, $h(80)$, $h(800)$, $h(8000)$ and $h(-2)$, $h(-20)$, $h(-200)$, $h(-2000)$

D) State a conclusion about $f(x)$ as x gets very large (and when x gets very negative). What function does $f(x)$ almost duplicate?

E) Make up number lines for $f(x)$ and $f'(x)$.

1) Specifically state the interval(s) where $f(x)$ is above the x-axis.

2) Specifically state the interval(s) where $f(x)$ is increasing.

3) Are your intervals for (1) and (2) the same? Why or Why not?

F) Determine, identify and justify the coordinates of any extrema of $f(x)$.

G) On a single axis...

1) Draw the line $x=3$ (dotted)

2) Draw the line $y=x+3$.

3) Plot the points from part (F).

4) Draw $f(x)$

5) Does your picture agree with your answers to part E (1) and (2)?

IV Given $f(x)=x^4-4x^3+3$ and $[-1,4]$

A) Find the coordinates of all extrema. (Justify, of course.)

B) Sketch $f(x)$, $f'(x)$, and $f''(x)$ on a single y-axis with 3 different x-axes.

V Given $f(x)$ is continuous at $x=a$. Only one of the following may be false. Determine which one may be false and draw a picture of a function which satisfies all of the others except that one you have picked.

i) $\lim_{x\to a} f(x)$ exists ii) $\lim_{x\to a} f(x)=f(a)$ iii) $f(a)$ and $f'(a)$ exist iv) $\lim_{x\to a^-} f(x)=\lim_{x\to a^+} f(x)$

VI The derivative of $f(x)=\frac{1}{3}x^4-\frac{1}{5}x^5$ attains its maximum value(s) at what value(s) of x?

VII The line $3x-4y=0$ is tangent in the first quadrant to $y=x^3+k$. Find k.

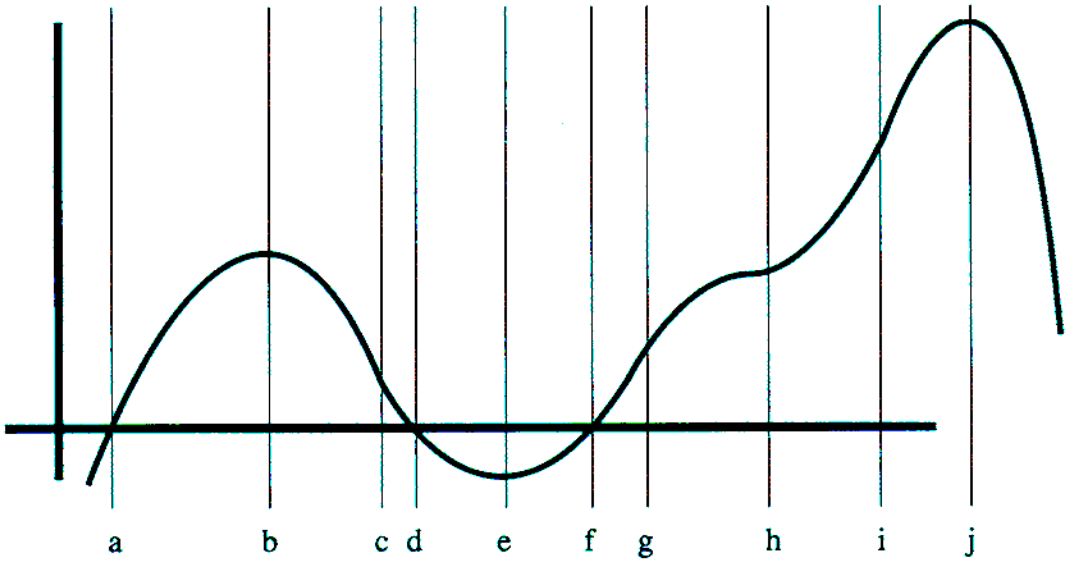
VIII Given $f(x)=3x^5-20x^3$. Sketch. Determine extrema. (Justify, of course)

IX Given $f(x)=x^3-x^2-4x+4$. Find the equation of the tangent line to $f(x)$ at $x=-1$.

X True or False (and explain): $f(x)=5x^4-x^5$ has two inflection points.

XI Using words only, explain the three step process required to prove that a function is continuous at a particular point. Give picture examples of functions which illustrate when one or more of the "parts of the definition of continuity" has been violated.

XII The following picture is the graph of $f'(x)$. Note it is *not* the graph of $f(x)$.
Draw the pictures of $f(x)$ and $f''(x)$.
Specifically state coordinates of significant items. Justify, of course.



Number Lines– Я –Us

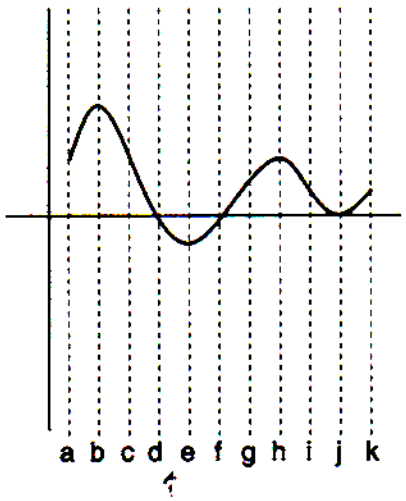
Mr. Mumaugh
Analysis Ch 3

I On the number line sheet provided, **mark** the f f' f'' number lines.

On separate paper, describe all max, min and PI for $f(x)$ and include proper justification.

(for example: $(e, f(e))$ max f' test or
$(a, f(a))$ min End Pt. increasing from it.)

a)



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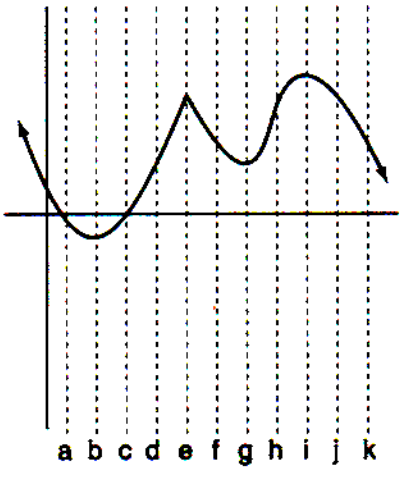
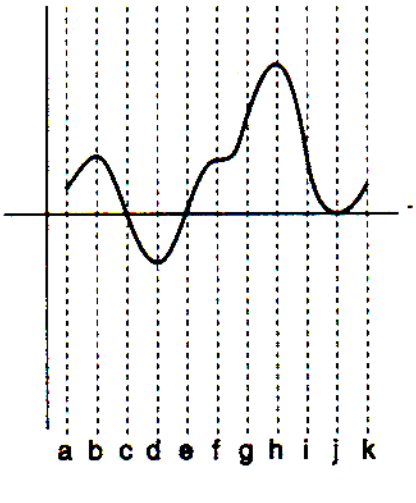


c)



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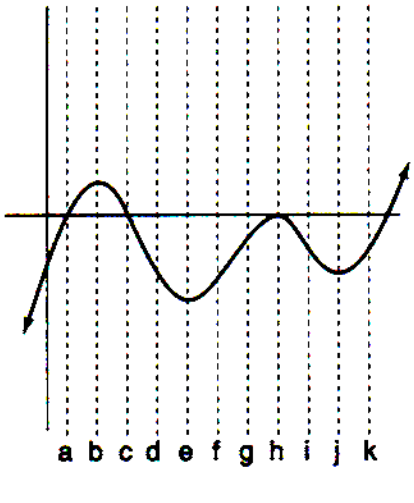

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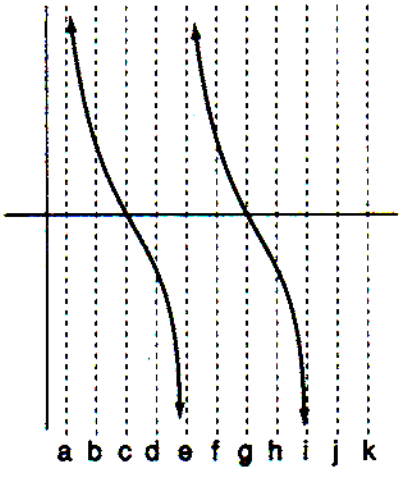
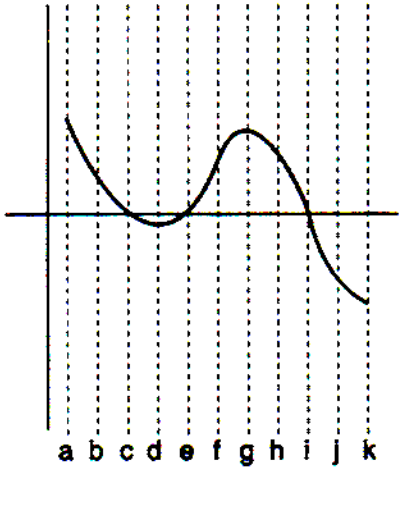


f)



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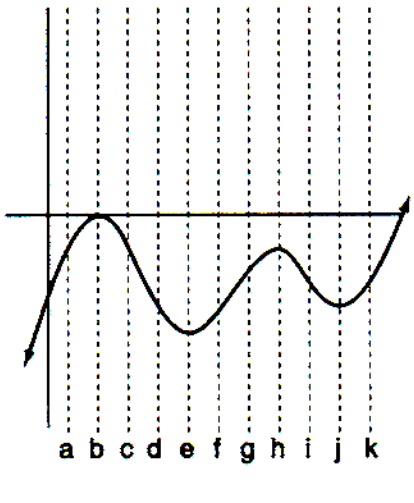

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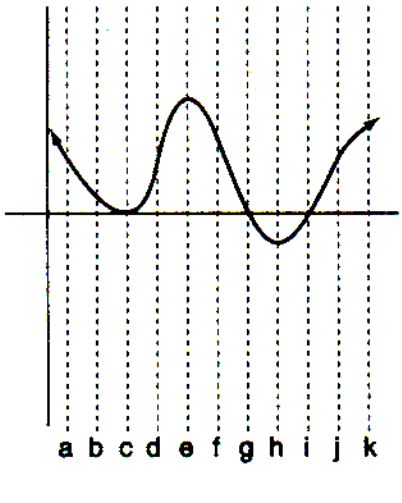

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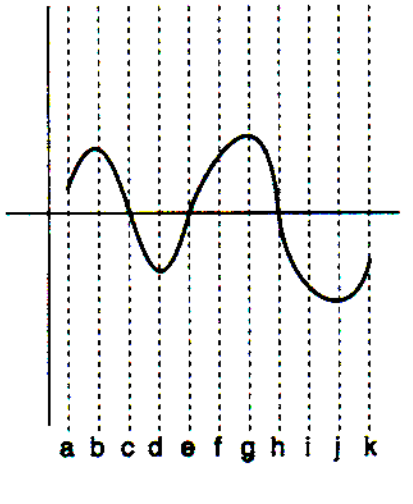
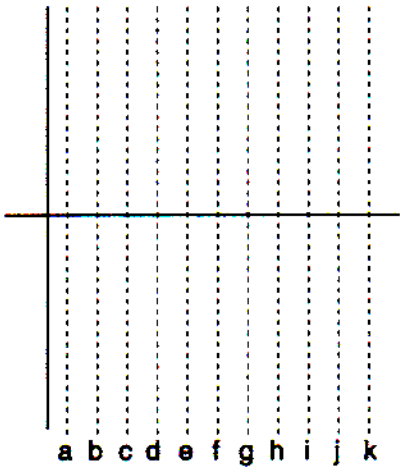


II Use the marked number lines for clues which will allow you to sketch ***only*** the graph of the function, $f(x)$.
On separate paper, describe all max, min and PI and include proper justification.
(for example: $(e, f(e))$ max f' test or
$(a, f(a))$ min End Pt. increasing from it.)

a)



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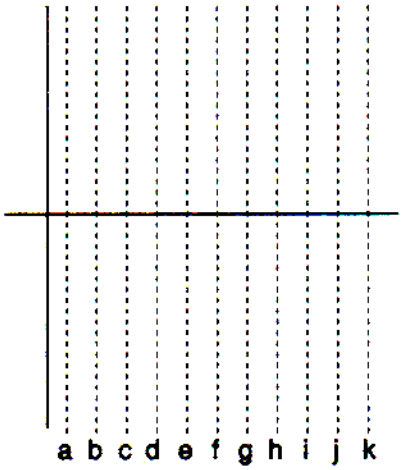

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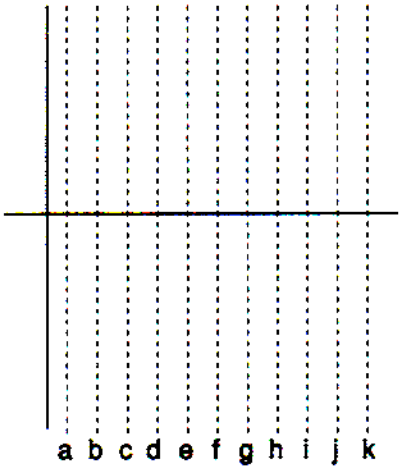

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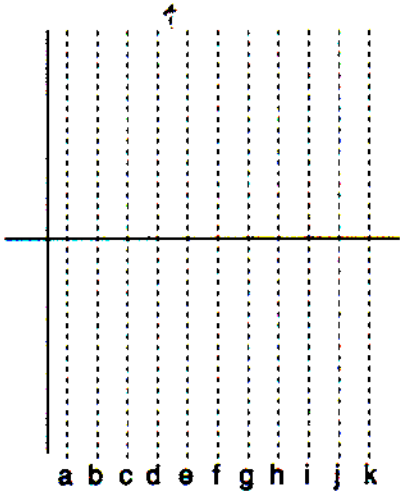

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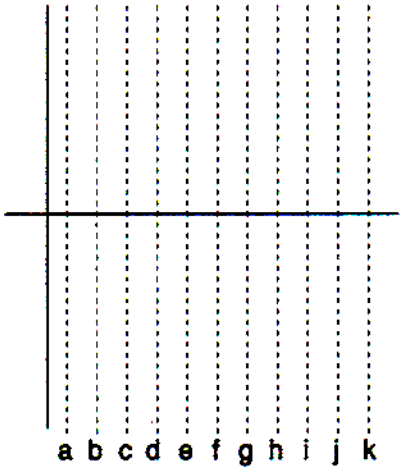

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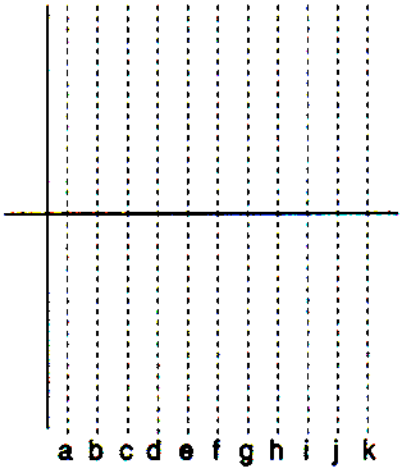

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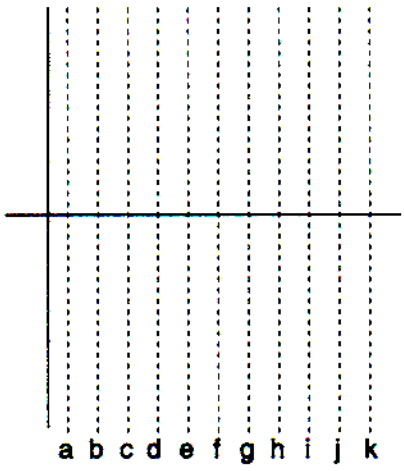

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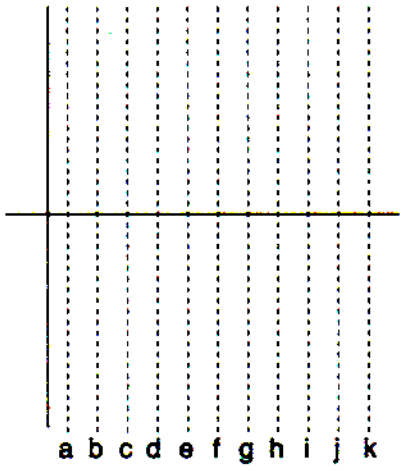

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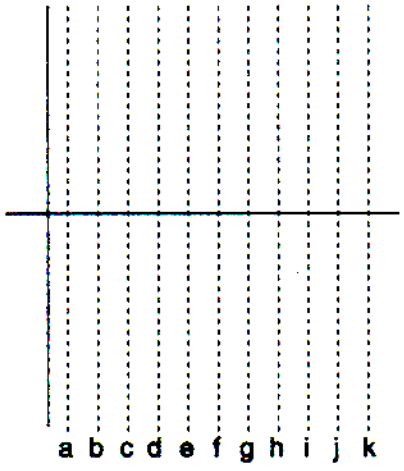
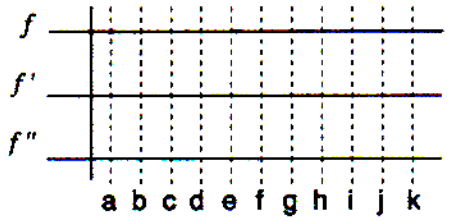


I continued ...

○ = a zero ⊗= Non Participating Point ×= DNE at that point

a)



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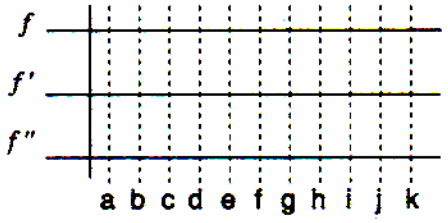

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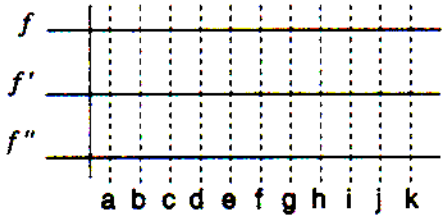

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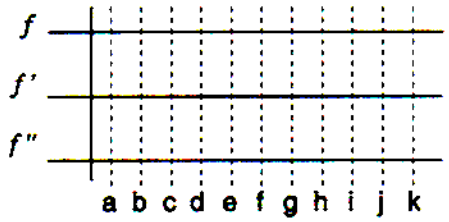

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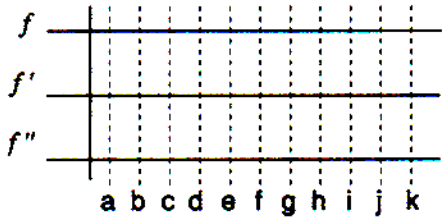

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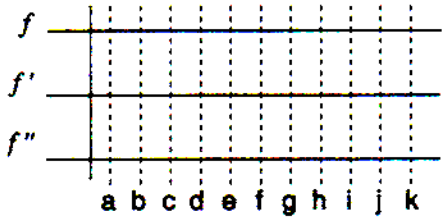

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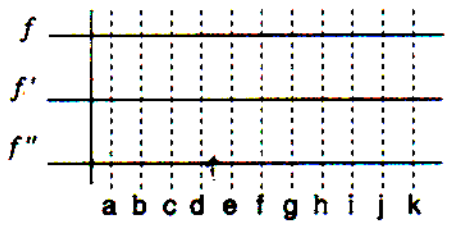

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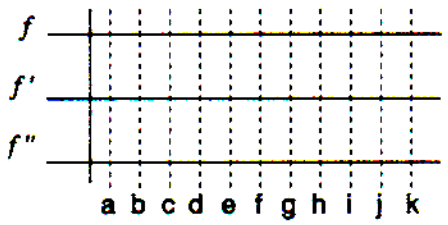

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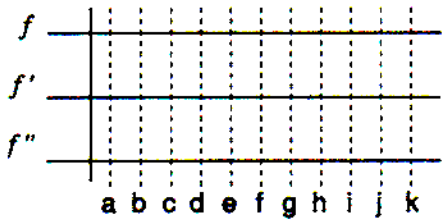
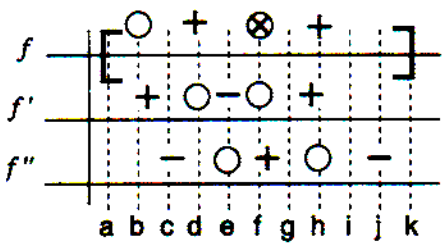


II continued...

a)



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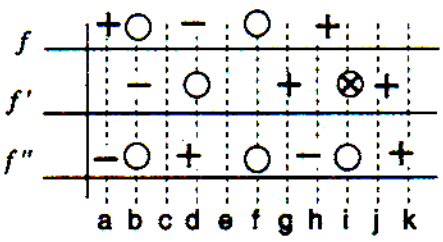

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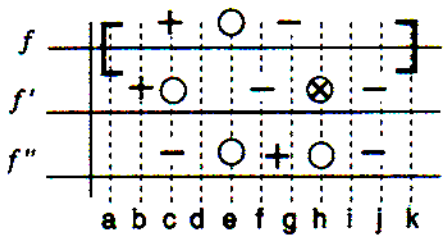

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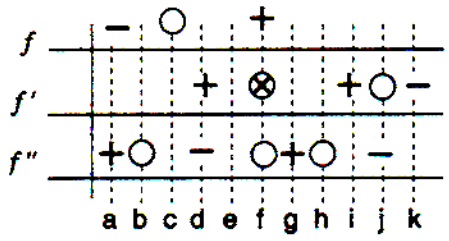

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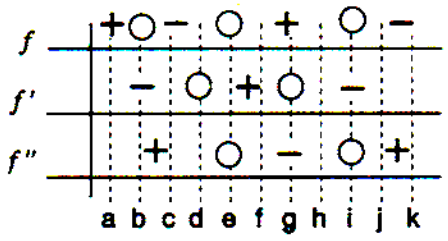

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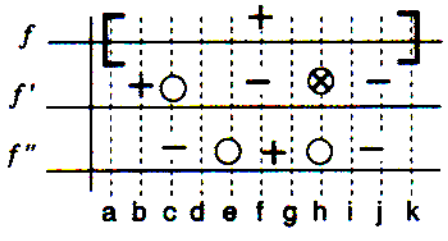

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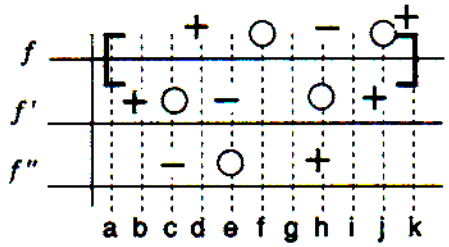

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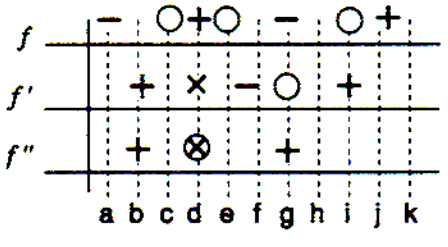
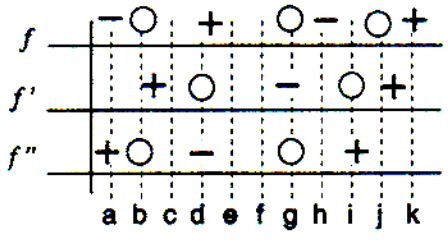


i)



unLIMITed Enjoyment

Honors Analysis

Mr. Mumaugh

I Find the following limits:

a) $\lim_{x\to 1}\dfrac{x-1}{\sqrt{x}-1}$ b) $\lim_{h\to 0}\dfrac{h^2}{6-\sqrt{36-h^2}}$ c) $\lim_{x\to 1}\dfrac{\frac{1}{x}-1}{x-1}$ d) $\lim_{x\to 1}\dfrac{\frac{1}{\sqrt{x}}-\frac{1}{\sqrt{2}}}{x+2}$

e) $\lim_{h\to 0}\dfrac{\sqrt{2+h}-\sqrt{2}}{h}$ f) $\lim_{x\to 1}\dfrac{\sqrt{x+3}-2}{x-1}$ g) $\lim_{h\to 0}\dfrac{(a+h)^2-a^2}{h}$ h) $\lim_{h\to 0}\dfrac{\sqrt{a+h}-\sqrt{a}}{h}$

i) $\lim_{x\to -2}(x+3)^{99}$ j) Find the limit **as h goes to zero** for problems 38, 39, 40 on page 11 in the text.

k) Clear parentheses: $(a^2-ab+b^2)(a+b)$ l) Clear parentheses: $(p-q)(p^2+pq+q^2)$

m) Show long division: $2x^2+5x-3\overline{)6x^3+19x^2+x-6}$ n) $\lim_{x\to -3}\dfrac{6x^3+19x^2+x-6}{2x^2+5x-3}$

o) $\lim_{x\to -\frac{2}{3}}\dfrac{6x^3+19x^2+x-6}{2x^2+5x-3}$ p) $\lim_{x\to \frac{1}{2}}\dfrac{2x^2+5x-3}{6x^3+19x^2+x-6}$

II Find the following limits.

a) $\lim_{x\to 0}\dfrac{\sqrt{2+x}-\sqrt{2}}{x}$ b) $\lim_{x\to 0}\dfrac{1-\sqrt{2x^2+x+1}}{x}$ c) $\lim_{x\to 2}\dfrac{\frac{1}{\sqrt{x^2}}-\frac{1}{2}}{x+2}$ d) $\lim_{x\to 1}\dfrac{x^2-2x-3}{x^2-1}$

e) $\lim_{x\to 1}\dfrac{x^2+2x-3}{x^2-1}$ f) $\lim_{h\to 0}\sqrt{1+\frac{1}{h}}-\sqrt{\frac{1}{h}}$ g) $\lim_{x\to -2}\dfrac{\frac{1}{\sqrt{x^2}}-\frac{1}{2}}{x+2}$ h) $\lim_{x\to 2}\dfrac{x^4-16}{x-2}$

i) $\lim_{h\to 0}\dfrac{(x+h)^2-x^2}{h}$ j) $\lim_{x\to 0}\dfrac{\sqrt[3]{x+1}-1}{x}$ k) $\lim_{x\to -2}\dfrac{\frac{1}{\sqrt{x^2}}-\frac{1}{2}}{x-2}$ l) $\lim_{h\to 0}\dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$

m) $\lim_{x\to 1}\left(\dfrac{1}{1-x^3}+\dfrac{1}{3(x-1)}\right)$ n) $\lim_{h\to 0}\left(\dfrac{(x+h)^3-x^3}{h}\right)$

III Find the limits:

a) $\lim_{x\to 2^-}\sqrt{2-x}$ b) $\lim_{x\to 0^+}\sqrt{x}$ c) $\lim_{x\to 0^-}\sqrt{x}$ d) $\lim_{x\downarrow 1}\dfrac{x-1}{|x-1|}$

e) $\lim_{x\uparrow 1}\dfrac{x-1}{|x-1|}$ f) $\lim_{x\to 1}\dfrac{x-1}{|x-1|}$ g) $\lim_{x\to 4^+}\sqrt{x-4}-2$ h) $\lim_{x\to 0}\dfrac{\sqrt{x^2+9}-3}{x^2}$

i) $\lim_{x\to 0}\dfrac{\sqrt{x+4}-2}{x}$ j) $\lim_{x\to 3}\sqrt{x-5}$ k) $\lim_{x\to 0}\left(\dfrac{3+2x}{x+5x^2}-\dfrac{3}{x}\right)$

l) Re-write as a piece meal function: $F(x)=\dfrac{x+|x|}{|x|}$ m) Graph $F(x)=\dfrac{x+|x|}{|x|}$ n) $\lim_{x\to 0}\dfrac{x+|x|}{|x|}$

n) Explain (in English) ***exactly*** (no examples, please) what is meant by: $\lim_{x\to a} f(x)=L$. Be sure to include how "left-hand limits" , "right-hand limits", and "limits" are related and how they are different